15

A wave is a periodic disturbance that travels through a medium (or vacuum). In general, a wave transports energy and momentum from one part of the medium (or vacuum) to the other part of it, without any bulk motion of the material of medium. All our communications essentially depend on transmission of signals through waves.

WAVES

|TOPIC 1|
Waves and Their Superposition

Waves occur when a system is disturbed from its equilibrium position and this disturbance travels or propagates from one region of the system to other. In a wave, both information and energy propagate (in the form of signals) from one point to another but there is no motion of matter as a whole through a medium.

Types of Waves

We usually deal with three types of waves as given below

Mechanical Wave

The waves requiring a material medium for their propagation are called mechanical waves. These waves are also called **elastic waves** because their propagation depend on the material media which possess elasticity and inertia. e.g. Water waves, sound waves and seismic waves, energy waves that travel through the earth's layers. These waves are governed by Newton's laws of motion and can exist only in a material medium such as water, air and rock, strings, etc.

Electromagnetic Wave

These types of waves travel in the form of oscillating electric and magnetic fields are called electromagnetic waves.

Electromagnetic waves do not require any material medium for their propagation and are also called **non-mechanical waves**.

CHAPTER CHECKLIST

- Waves and Their Superposition
- Types of Waves
- Characteristics of Wave Motion
- Transverse & Longitudinal Wave
- Displacement Relation in a Progressive Wave
- Speed of a Travelling Wave
- Principle of Superposition of Waves
- Reflection of Waves
- Standing Waves and Normal Modes
- Vibrations of Air Columns
- Beats

e.g. Visible and ultraviolet light, X-rays, radio and television waves, microwaves etc. All the electromagnetic waves travel through the vacuum at the same speed c given as

$$c = 29{,}97{,}92{,}458 \text{ m/s} \qquad \text{[speed of light]}$$
$$\simeq 3 \times 10^8 \text{ m/s}$$

Matter Wave

These types of waves are associated with microscopic particles i.e. electrons, protons and other fundamental particles and even atoms and molecules when they are in motion are called matter waves or de-Broglie waves.

e.g. electron microscope is associated with electrons present in matter wave.

> ***Dual Nature of Light***
>
> - Light wave is an electromagnetic wave. Every radiation that we get from the sun comes under electromagnetic waves. Actually, light shows dual nature. Light behaves as wave, as well as particle.
> - In the same way, a moving particle could behave as wave. Matter wave is the wave associated with a moving particle.

Characteristics of Wave Motion

(i) In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.

(ii) The energy is transferred from one place to another without any actual transfer of the particles of the medium.

(iii) There is a continuous phase difference between two successive particles because each particle receives disturbance a little later than its preceding particle.

(iv) The velocity with which a wave travels is different from the velocity of the particles with which they vibrate simple harmonically about their mean positions.

(v) In a given medium, the wave velocity remains constant, while the particle velocity changes continuously during its vibration about the mean position.

The particle's velocity is maximum at the mean position and zero at the extreme position.

(vi) For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity as well as friction.

Spring Model for the Propagation of a Wave

Consider a collection of springs connected to one another and one end is fixed to a rigid support. If the spring at one end is pulled suddenly and released, the disturbance travels to the other end.

A collection of springs connected to each other

The reason behind this is when the first spring is pulled, it gets stretched. Due to elasticity, a restoring force is developed in the first spring.

This restoring force brings the first spring back to its mean position and stretches the second spring and so on. Thus, the disturbance moves from one end to the other, but each spring oscillating about its equilibrium.

Propagation of Sound Wave through Air

Consider the small region of air as a spring. It is connected to the neighbouring regions or springs. As sound waves pass through air, it compresses and expands a small region of air. This causes a change in density and pressure of that region.

According to Boyle's law,

$$\text{Change in pressure } (\Delta p) \propto \frac{1}{\text{Change in volume } (\Delta V)}$$

$$\Rightarrow \qquad (\Delta p) \propto (\Delta \rho)$$

As the pressure is force per unit area, so a restoring force is proportional to the disturbance or change in density is developed just like in an extended or compressed spring.

If a region is compressed, its molecules tend to move out to the adjacent region, thereby, increasing the density or creating compression in that region. The air in the first region undergoes rarefaction due to decrease in its density in the first region.

But, if a region is comparatively rarefied, the surrounding air will rush in making the rarefaction move to the adjoining region. Thus, the compression or rarefaction moves from one region to another, making the propagation of a disturbance possible in air.

Propagation of Sound Wave in a Solid

In a crystalline solid, various atoms can be considered as end points, with springs connected between pairs of them. In this, each atom or group of atoms is in equilibrium due to forces from the surrounding atoms because the forces exerted by the other atoms are cancelled out.

When the sound wave propagates, the atom is displaced from its equilibrium position and a restoring force is developed. This disturbance produced by the force travels to the next atom and so on. Thus, the wave propagates through the solid.

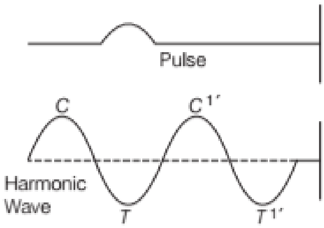
TRANSVERSE WAVE

Transverse wave motion is that wave motion in which the individual particles of the medium execute simple harmonic motion about their mean positions in a direction perpendicular to the direction of propagation of the wave. The wave itself is known as **transverse wave.**

e.g. Waves in string, ripples on the surface of water. Movement of string of stringed musical instruments like sitar, guitar.

Illustration for Transverse Wave

Consider a horizontal spring with its one end held in the hand and other end fixed to a rigid support as shown in figure.



Transverse wave along the stretched string

If we give its free end a smart upward jerk, an upward kink or pulse is created there, which travels along the string towards the fixed end. Hence, if we continuously give up and down jerks to the free end of the string, the string successively undergoes a disturbance about its mean position and a number of sinusoidal waves begin to travel along the string. So that, the waves in the string are transverse in nature.

The points (C, C', ...) called **crest** show the position of maximum displacement in the positive direction i.e. upward direction.

The points (T, T', ...) called **trough** show the position of maximum displacement in the negative direction i.e. downward direction.

Media for Transverse Waves

In transverse waves, the particle's motion is perpendicular to the direction of propagation of the wave. As the wave propagates, each element of the medium undergoes a **shearing strain**.

Transverse waves can, therefore, be propagated only in those media which can sustain **shearing stress**, such as solids and springs but not in fluids.

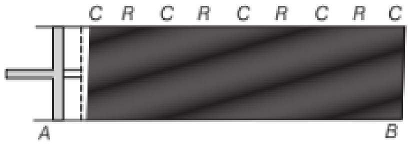
LONGITUDINAL WAVE

The waves in which the individual particles of the medium execute simple harmonic motion about their mean positions along the direction of propagation of the wave are called **longitudinal waves**.

e.g. Waves in spring, sound waves, vibration of air column in organ pipes, etc.

Illustration for Longitudinal Wave

Consider the production of waves in a long pipe filled with air has a piston at one end.



Production of Longitudinal waves in a cylinder by moving a piston back and forth.

If we push the piston suddenly towards right, a small layer of air just near the piston head is compressed and after being compressed, this layer moves towards right and compresses the next layer and so on, the compression reaches the other end. Thus, the motion of air and the changes in air pressure (due to the change in density of the medium in compression) travel towards the right as a pulse.

Now, if the piston is pulled towards left, there is rarefied in the layer adjacent to the piston occured resulting in the fall of pressure. The air from the next layer moves to restore pressure. Consequently, the next layer is rarefied. In this way, a pulse of rarefaction moves towards right.

Due to continuous pushing and pulling of the piston in a simple harmonic manner, a sinusoidal wave travels in the cylinder in the form of **compressions** and **rarefactions**, (marked C, R, C, R, etc.) resulting a temporary reduction in volume and consequent increase in density.

As the motion of the oscillation of the elements of air is parallel to the direction of propagation of wave along the pipe, the sound waves produced in air are **longitudinal** waves.

A compression is a region of the medium in which the distance between any two consecutive particles of the medium is less than the normal distance. Hence, temporarily there will be decrease in volume, increase in density.

A rarefaction is a region of the medium in which the distance between any two consecutive particles of the medium is more than the normal distance. Consequently, there will be increase in volume and decrease in density of the medium temporarily.

Media for Longitudinal Waves

Fluids as well as solids can sustain compressive strain, therefore, longitudinal waves can propagate in all elastic media.

Few characteristics of a media for propagation of wave

(i) A medium like a steel bar, both transverse and longitudinal waves can propagate while air can sustain only longitudinal waves.

(ii) Transverse and longitudinal waves travel with different speeds, when they propagate through the same medium.

(iii) In a wave motion, only the disturbance travels through a medium. The medium does not travel with the disturbance.

(iv) Waves may be one-dimensional, two-dimensional or three-dimensional. Mode of dimension will be according to the propagation of energy in one, two or three dimension.

(v) Transverse waves along a string are one-dimensional, ripples on water surface are two-dimensional and sound waves produced from a point source are three-dimensional.

EXAMPLE |1| Identifying the Waves

Given below are some examples of wave motion. State in each case if the wave motion is transverse, longitudinal or a combination of both.

(i) Motion of a kink in a longitudinal spring produced by displacing one end of the spring side ways.

(ii) Waves produced in a cylinder containing a liquid by moving its piston back and forth.

(iii) Waves produced by a motorboat sailing in water.

(iv) Ultrasonic waves in air produced by a vibrating quartz crystal. [NCERT]

Sol. (i) Transverse and longitudinal

(ii) Longitudinal

(iii) Transverse and longitudinal

(iv) Longitudinal

Displacement Relation in a Progressive Wave

To describe travelling wave mathematically, we need position x and time t. Consider the wave travelling in positive x-direction. The displacement $y(x, t)$ denotes the transverse displacement of the element at position x at time t and is given by

$$\boxed{\text{Displacement, } y(x, t) = a\sin(kx - \omega t + \phi)} \quad \ldots\text{(i)}$$

The above equation can also be represented by using linear combination of sine and cosine functions such as

$$y(x,t) = A\sin(kx - \omega t) + B\cos(kx - \omega t)$$

where, $$a = \sqrt{A^2 + B^2} \text{ and } \phi = \tan^{-1}\left(\frac{B}{A}\right) \quad \ldots\text{(ii)}$$

Clearly from above equation, we can say that the constituents of the medium at different positions execute SHM.

As the above equation is written in terms of position x, it can be used to find the displacements of all the elements of the string as a function of time.

Thus, it can tell us the shape of the wave at any given time and how that shape changes as the wave moves along the string and hence, how the wave progress.

The wave travelling in the negative direction of X-axis can be, represented by

$$y(x,t) = a\sin(kx + \omega t + \phi) \quad \ldots\text{(iii)}$$

where,

$y(x,t)$ = displacement of vibrating element or particle as a function of position x and time t.

a = amplitude of the vibrating element or particle

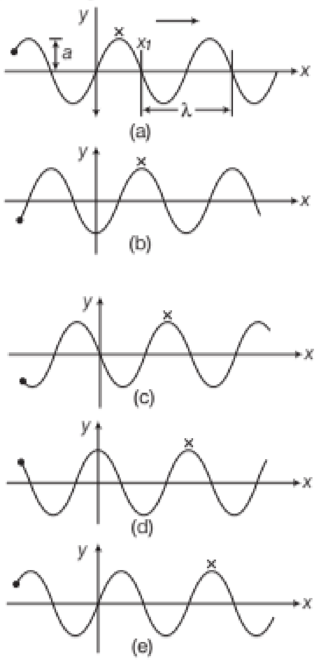
ω = angular frequency of the wave

x = position of vibrating element

k = angular wave number.

$(kx - \omega t + \phi)$ = phase of a vibrating element at time t and position x. ϕ is phase constant at $x = 0$, $t = 0$.

The figure below shows the plots of Eq. (i) for different values of time and position. In the plot, crest is the point of maximum positive displacement and trough is the point of maximum negative displacement.



Plots for a harmonic wave travelling in positive x-direction at different time

To analyse the motion, we have represented dot (•) and cross (×) in the figure.

Amplitude and Phase

The magnitude of the maximum displacement of the particles from their equilibrium position, as a wave passes through them is known as **amplitude** of the particle.

In Eq. (i), the displacement $y(x, t)$ varies between a and $-a$. Here, a represents the maximum displacement of the constituents of the medium from their mean position.

The displacement y may be positive or negative but a is positive. It is called the **amplitude** of the particle. The quantity $(kx - \omega t + \phi)$ is called the **phase** of the wave which appears as the argument of the sine or cosine function.

Wavelength and Angular Wave Number

The minimum distance between two particles having the same phase is called the **wavelength** of the wave, usually denoted by λ.

Or

The distance travelled by a wave during the time in which any particle of the medium completes one vibration about its mean position is known as **wavelength** of the wave.

As, $\quad y(x, t) = a \sin(kx - \omega t + \phi)$

Taking $\phi = 0$ in the displacement at $t = 0$, $y(x, 0) = a \sin kx$

Because, the sine function is having period 2π.

$$\therefore \quad \sin kx = \sin(kx + 2n\pi) = \sin k\left(x + \frac{2n\pi}{k}\right)$$

The displacement will be same at points x and $x + \dfrac{2n\pi}{k}$ with $n = 1, 2, 3, \ldots$. The least distance between two points with same phase will be

$$\lambda = \frac{2\pi}{k} + x - x \text{ (for } n = 1)$$

$$\boxed{\text{Wavelength, } \lambda = \frac{2\pi}{k}}$$

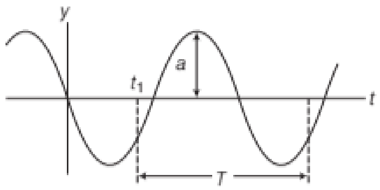
$$\Rightarrow \quad \text{Angular wave number, } k = \frac{2\pi}{\lambda} \text{ rad m}^{-1}.$$

where, k is called **angular wave number**. Hence, the angular wave number also known as **propagation constant** is 2π times the number of waves that can be accommodated per unit length.

Period, Angular Frequency and Frequency

Figure below shows a sinusoidal plot. It describes the shape as well as displacement. We are monitoring the motion of the element by putting $\phi = 0$ and $x = 0$ in equation given as

$$y(x, t) = a \sin(kx - \omega t + \phi)$$



Graph for displacement time period, amplitude and frequency

We will get $y(0, t) = a \sin(-\omega t) = -a \sin \omega t$

Time period

The time taken by a wave to travel a distance equal to one wavelength is known as the **time period** of wave.

If period of sine function is T measured from an arbitrary time t_1, then

$$\Rightarrow \quad -a \sin \omega t_1 = -a \sin \omega (t_1 + T)$$
$$= -a \sin (\omega t_1 + \omega T)$$

Because, sine function repeats after every 2π.

$$\therefore \quad \omega T = 2\pi$$

$$\Rightarrow \quad \boxed{\text{Angular frequency, } \omega = \frac{2\pi}{T}}$$

where, ω is called the **angular frequency**, SI unit is rad s^{-1}.

The number of complete vibrations or oscillations produced by a wave in one second is known as the **frequency** of the wave. The frequency ν of a wave is defined as $\dfrac{1}{T}$ and is related to the angular frequency ω by

$$\boxed{\text{Frequency, } \nu = \frac{1}{T} = \frac{\omega}{2\pi}}$$

where, $\nu =$ frequency of the wave

ν is usually measured in hertz (Hz).

The relation between the wavelength λ and velocity v of the wave is given as $\quad \lambda = \dfrac{v}{n}$

Note

The frequency of a wave will remain unchanged during any type of interaction that the wave suffers because it depends only on the frequency of the source.

EXAMPLE |2| Formation of the Progressive Wave

If a progressive wave travelling in positive x-direction having the amplitude of 6 cm, frequency 200 Hz and velocity is 400 m/s, then write the equation of that progressive wave.

Sol. Given, $A = 6\,\text{cm} = 0.06\,\text{m}$

$$\nu = 200\,\text{Hz}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{v/\nu} \qquad \left[\because \lambda = \frac{v}{\nu}\right]$$

$$k = \frac{2\pi\nu}{v} = \frac{2\pi \times 200}{400} = \pi\,\text{m}^{-1}$$

$$\omega = 2\pi\nu = 2\pi \times 200 = 400\pi\ \text{rad/s}$$

The standard equation of the progressive wave is

$$y(x, t) = A\sin(kx - \omega t)$$

put the values to get the equation

$$y(x, t) = 0.06\sin(\pi x - 400\pi t)\ \text{m}$$

EXAMPLE |3| Wave in a String

A wave travelling along a string is described by $y(x,t) = 0.005\sin(80.0x - 3.0t)$ in which the numerical constants are in SI units (0.005 m, 80.0 rad m^{-1} and 3.0 rad s^{-1}). Calculate

(i) the amplitude of particle,

(ii) the wavelength and

(iii) the period and frequency of the wave. Also, calculate displacement y of the particle at a distance $x = 30.0$ cm and time $t = 20$ s ? **[NCERT]**

Sol. Given, $y(x,t) = 0.005\ \sin(80.0x - 3.0t)$

Then, $y(x,t) = a\sin(kx - \omega t)$

Now, compare the given equation with standard equation to find out all the physical quantities.

$$a = 0.005\ \text{m}$$

$$k = 80.0\ \text{rad/m}$$

$$\omega = 3.0\ \text{rad/s}$$

The physical quantities by using the given fundamental physical quantities.

(i) Amplitude, $a = 0.005\ \text{m} = 5\ \text{mm}$

(ii) Wavelength, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{80} = 7.85\text{cm}$

(iii) Time period, $T = 2\pi/\omega = \frac{2\pi}{3} = 2.09\ \text{s}$

and frequency, $\nu = \frac{1}{T} = \frac{1}{2.09}$

$$= 0.478 \simeq 0.48\,\text{Hz}$$

Now, the displacement y of the particle at a distance.

$x = 30.0$ cm $= 0.3$ m and time $t = 20$ s

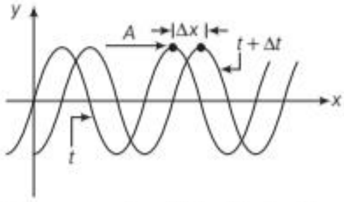
$$y(0.3, 20) = 0.005\ \sin(80 \times 0.3 - 3.0 \times 20)$$

$$y(0.3, 20) = 0.005\ \sin(24 - 60)$$

$$= 0.00495\ \text{m} \simeq 5\ \text{mm}$$

Speed of a Travelling Wave

We can fix our attention on any particular point on the wave to determine the speed of propagation. It is convenient to look at the motion of the crest. In the figure shown below, we have shown the shape of the wave at two instants of time t and $t + \Delta t$ and the wave pattern shifted by distance Δx.



Wave moves to right with velocity v

As shown in above waveform, each point of the moving waveform, such as point A marked on a peak, retain its displacement y.

Here, it must be noted that the points on the string do not retain this displacement but points on the waveform do. As point A retains its displacement as it moves, the phase in the equation of the displacement $y(x,t)$ is constant. Thus,

$\Rightarrow \qquad kx - \omega t = \text{constant}$

$\Rightarrow$ If x changed to $x + \Delta x$ and t to $t + \Delta t$

$\Rightarrow \qquad kx - \omega t = k(x + \Delta x) - \omega(t + \Delta t)$

or $\qquad k\Delta x = \omega \Delta t \Rightarrow \frac{\Delta x}{\Delta t} = \frac{\omega}{k} = v$

$\Rightarrow \qquad v = \frac{2\pi\nu}{k} = \nu \times \left(\frac{2\pi}{k}\right) = \nu\lambda$

$\therefore \qquad \boxed{\text{Speed of travelling wave, } v = \nu\lambda = \frac{\lambda}{T}}$

The above equation is general relation for all progressive waves, shows that the wave pattern travels a distance equal to the wavelength of the wave.

Speed of a Transverse Wave on Stretched String

The speed of a mechanical wave is related with the restoring force set up in the medium due to the disturbance produced. For wave on a string, it will be provided by the tension (T).

We will use dimensional method to get the equation for speed.

Combining dimensions of mass per unit length (or linear mass density) μ of the string as $[\text{ML}^{-1}]$ and tension T of the string as $[\text{MLT}^{-2}]$, we get

$$\frac{[\text{MLT}^{-2}]}{[\text{ML}^{-1}]} = [\text{L}^2\text{T}^{-2}]$$

As dimension for v is $[LT^{-1}]$

$$\boxed{\text{Speed of a transverse wave, } v = c\sqrt{\frac{T}{\mu}}}$$

where, c is a dimensionless constant, c is turned out to be 1.

$$\Rightarrow \qquad v = \sqrt{\frac{T}{\mu}}$$

The speed of the wave along a stretched ideal string depends only on the tension and the linear mass density of the string and does not depend on the frequency of the wave.

The frequency of the wave is determined by the source that generates the wave. The wavelength is then given in the form

$$\lambda = \frac{v}{\nu}$$

EXAMPLE |4| Stretched Wire

A wave moves with speed 300 m/s on a wire which is under tension of 400 N. Find how much tension must be changed to increase the speed to 315 m/s?

Sol. We know speed, $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad \frac{dv}{dT} = \frac{1}{2\sqrt{\mu T}}$$

$$\Rightarrow \quad \frac{dv}{v} = \frac{1}{2}\frac{dT}{T} \qquad \left[\text{putting } \sqrt{\mu} = \frac{\sqrt{T}}{V}\right]$$

$$\Rightarrow \quad dT = (2T)\frac{dv}{v}$$

$$= 2 \times 400 \times \left(\frac{315 - 300}{300}\right)$$

$$= \frac{2 \times 4}{3} \times 15 = 2 \times 4 \times 5 = 40 \text{ N}$$

Hence, tension should be increased by 40 N.

or

Alternate Method

As speed $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \qquad [\mu \text{ is same}]$$

$$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{315}{300}\right)^2 = \left(\frac{300+15}{300}\right)^2$$

$$\frac{T_2}{T_1} = \left(1 + \frac{15}{300}\right)^2 = \left(1 + \frac{1}{20}\right)^2$$

$$\frac{T_2}{T_1} = 1 + 2 \times \frac{1}{20} \qquad [\text{use Binomial theorem}]$$

$$\frac{T_2}{T_1} = 1 + \frac{1}{10}$$

$$\frac{T_2}{T_1} - 1 = \frac{1}{10} \Rightarrow \frac{T_2 - T_1}{T_1} = \frac{1}{10}$$

$$\Delta T = \frac{1}{10} \times T_1 = \frac{1}{10} \times 400 = 40 \text{ N}$$

Hence, tension should be increased by 40 N.

(Previous method is not familiar for the student, so, the given solution gives more comfort to the student).

EXAMPLE |5| Tension in a Cord

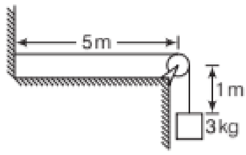
A uniform cord have a mass 0.2 kg and length 6 m. If tension is maintained in the cord by suspending a mass of 3 kg from one end, then find out the speed of a pulse on this cord. Also find the time, it takes the pulse to travel from the wall to the pulley.

Sol. Given, Mass of a suspended block, $m = 3$ kg

Mass of the cord = 0.2 kg

Length of the cord = 6 m

Speed of the cord = ?



Tension, T = weight of mass of 3 kg = $m \times g$

$= 3 \times 9.8 = 29.4$ N

$$\mu = \frac{m}{l} = \frac{0.2}{6}$$

$= 0.033$ kg/m

As we know the wave speed or speed of a pulse on the cord is given by

$$v = \sqrt{\frac{T}{\mu}}$$

Put the values to get the speed of the pulse.

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{29.4}{0.033}} \simeq 29.85 \text{ m/s}$$

Thus, the time for this speed to travel from the wall to pulley i.e. a distance of 5 m.

$$\text{Time} = \frac{5}{29.85} = 0.168 \text{ s}$$

Speed of a Longitudinal Wave

(Speed of Sound)

In a longitudinal wave, the constituents of the medium oscillate forward and backward in the direction of wave propagation. In case of sound wave, it will be compression and rarefactions formed in air.

The Bulk modulus of the medium,

$$B = -\frac{\Delta p}{\Delta V / V}.$$

Where, Δp = change in pressure,

ΔV = change in volume

V = initial volume

B has a SI unit Pa.

Combining dimensions of $B[ML^{-1}T^{-2}]$ and density $\rho[ML^{-3}]$, we will get

$$\frac{[ML^{-1}T^{-2}]}{[ML^{-3}]} = [L^2T^{-2}]$$

$$\Rightarrow \qquad v = c\sqrt{\frac{B}{\rho}}$$

By experiment, $c = 1 \Rightarrow v = \sqrt{\frac{B}{\rho}}$

For a linear medium like a solid bar, the modulus of elasticity will be Young's modulus (Y). By analysing the dimensional analysis as previous the speed of longitudinal waves in a solid bar is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

where, Y is the Young's modulus of the material of the bar and ρ is density. Table below gives the speed of sound in some media.

Speed of Sound in Different Media

Medium	Speed (m s^{-1})
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater	1522
Solids	
Aluminium	6420
Copper	3560
Steel	5941
Granite	6000
Vulcanised rubber	54

Speed of Mechanical Waves

The speed of a mechanical wave, in a medium, depends upon the properties of the medium. For given type of waves (in a given medium), this speed, however does not change with a change in the characteristics (amplitude, wavelength or frequency) of the wave. e.g. the speeds of different types of sound waves, infrasonic, audible or ultrasonic, in a given medium, are all given by

$$\boxed{\text{Speed of mechanical waves, } v = \sqrt{\frac{E}{\rho}}}$$

where, E is the (appropriate) elastic constant and ρ is the density of the medium. The speed is thus, dependent only on the properties of the medium.

Speed of Electromagnetic Waves

The speed of propagation of electromagnetic waves is also determined by the characteristics of the medium through which they are propagating. Electromagnetic waves, as we know, are a combination of the oscillations of electric and magnetic fields in mutually perpendicular directions. The relevant properties of the medium, determining their speed of propagation, are the permittivity and the permeability of the medium.

In vacuum, the speed of propagation (c) of all types of electromagnetic waves, is given by $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Here, μ_0 = permeability of vacuum and ε_0 = permittivity of vacuum.

In any other material medium, the speed of propagation (v) of electromagnetic waves, is given by

$$\boxed{\text{Speed of electromagnetic waves, } v = \frac{1}{\sqrt{\mu\varepsilon}}}$$

where, μ = permeability of the medium and ε = permittivity of the medium. It however, turns out that the values of μ and ε, of a material medium, are not quite the same for electromagnetic waves of different frequencies.

EXAMPLE |6| Speed of Sound in Aluminium

An aluminium rod of length 90 cm and of mass is clamped at its mid-point and is set into longitudinal vibrations by stroking it with resined cloth. Assume that the rod vibrates in its fundamental mode of vibration. The density of aluminium is 2.6 g/cm^3 and its Young's modulus is 7.80×10^{10} N/m^2. Find the speed of the sound in aluminium. If the wavelength is 180 m then, find frequency of vibration.

Sol. Given, Young's modulus, $Y = 7.80 \times 10^{10}\ \text{N/m}^2$

Density of aluminium, $\rho = 2.6\ \text{g/cm}^3 = \dfrac{2.6 \times (100)^3}{1000}\ \text{kg/m}^3$

$$\rho = 2600\ \text{kg/m}^3$$

Wavelength, $\lambda = 180$ m

Speed of the sound in aluminium,

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7.80 \times 10^{10}\ \text{N/m}^2}{2600\ \text{kg/m}^3}}$$

$$v = 5477\ \text{m/s}$$

As, $\lambda = \dfrac{v}{\nu} \Rightarrow \nu = \dfrac{v}{\lambda} = \dfrac{5477\ \text{m/s}}{180\ \text{m}} = 30.42\ \text{Hz}$

Liquids and solids generally have higher speeds of sound than in gases.

Speed of the Longitudinal Wave in an Ideal Gas

For an ideal gas, we know that $pV = Nk_BT$

where, p is pressure, V is volume, N is molecular density, k_B is Boltzmann constant and T is the temperature on absolute scale (Kelvin).

For isothermal change, $\Delta T = 0$

$$\Rightarrow \quad V\Delta p + p\Delta V = 0 \quad \Rightarrow \quad -\frac{\Delta p}{\Delta V/V} = p$$

As, $$B = -\frac{\Delta p}{\Delta V/V}$$

$\therefore$ $$B = p$$

Hence, $$v = \sqrt{\frac{B}{\rho}}$$

or $$\boxed{\text{Speed of longitudinal waves in an ideal gas, } v = \sqrt{\frac{p}{\rho}}}$$

This is known as **Newton's formula.**

EXAMPLE |7| Moving with the Speed of Sound

Estimate the speed of sound in air at STP. The mass of 1 mole of air is 29.0×10^{-3} kg. **[NCERT]**

Sol. We know that volume of any gas at STP is 22.4 litre.

Density of air at STP is

$$\rho_0 = \left(\frac{\text{mass}}{\text{volume at STP}}\right)_{\text{for one mole of air}}$$

$$= \frac{29.0 \times 10^{-3}\ \text{kg}}{22.4 \times 10^{-3}} = 1.29\ \text{kgm}^{-3}$$

According to Newton's formula,

$$\text{Speed of sound, } v = \left[\frac{1.01 \times 10^5\ \text{Nm}^{-2}}{1.29\ \text{kgm}^{-3}}\right]^{1/2} = 280\ \text{ms}^{-1}$$

Laplace Correction

The result obtained as the speed of sound in air at STP is 280 ms^{-1} which is about 15% smaller as compared to the experimental value of 331 ms^{-1}. The mistake in the formula was pointed out by Laplace and he told that the changes in pressure and volume of a gas, when sound waves are propagated through it, are not isothermal, but adiabatic.

Hence, we will apply

$$pV^{\gamma} = \text{constant}$$

$$\Rightarrow \quad p\gamma V^{\gamma-1}\Delta V + V^{\gamma}\Delta p = 0$$

The adiabatic bulk modulus,

$$B_{\text{ad}} = -\frac{\Delta p}{\Delta V/V} = \gamma p$$

where, γ is the ratio of two specific heats C_p/C_V. The speed of sound is therefore, given by,

$$\boxed{\text{Speed of sound, } v = \sqrt{\frac{\gamma p}{\rho}}}$$

This modification of Newton's formula is referred to as the **Laplace correction.** For air, $\gamma = 7/5$.

By this formula, we will get a value of 331.3 ms^{-1}, which agree with the practical value.

Note

- Speed of longitudinal wave in term of temperature of the gas

$$V = \sqrt{\frac{\gamma RT}{M}}$$

where, T is temperature of the gas
M is molar mass of the gas

- According to the standard gas equation

$$pV = RT$$

$$\Rightarrow \quad p = \frac{RT}{V} \quad \text{...(i)}$$

$\therefore$ Speed of sound $v = \sqrt{\dfrac{\gamma p}{\rho}} = \sqrt{\dfrac{\gamma RT}{\rho \times V}} = \sqrt{\dfrac{\gamma RT}{M}}$

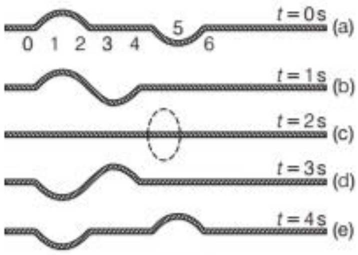
Substituting value of ρ, where $\rho \times V = M$

PRINCIPLE OF SUPERPOSITION OF WAVES

This principle states that when two or more pulses overlap, the resultant displacement is the algebraic sum of the displacements due to each pulse.

Consider the two waves travelling together along the same stretched string in opposite directions.

They meet and pass through each other, and move on independently as shown in figure.



Superposition of two waves

In the graph (c), the displacement due to the two pulses have exactly cancelled each other and there is zero displacement throughout.

Three important applications of superposition principle are:

1. Stationary waves
2. Beats
3. Interference of waves

Interference of Wave

Wave interference is the phenomenon that occurs when the two or more waves pass through the same medium. The interference of waves causes the medium to take on a shape that results from the net effect of the two individual waves upon the particles of the medium.

Let the two waves disturbances in the medium having displacements $y_1(x,t)$ and $y_2(x,t)$.

Then, the net displacement $y(x, t)$ is

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

For n waves superimposing with each other

$$y = f_1(x - vt) + f_2(x - vt) + \ldots + f_n(x - vt)$$
$$= \sum_{i=1}^{n} f_i(x - vt)$$

Here, function is represented for the moving waves. This principle of superposition is basic to the phenomenon of interference.

To illustrate the principle, we will consider two harmonic travelling waves having the same angular frequency ω and wave number $k = 2\pi/\lambda$. Let us assume that their amplitudes are identical.

If the two waves are out of phase by a phase constant ϕ, then the two waves are represented as

$$y_1(x, t) = a\sin(kx - \omega t)$$

and $$y_2(x, t) = a\sin(kx - \omega t + \phi)$$

By the principle of superposition, the net displacement of the resultant wave

$$y(x, t) = y_1(x, t) + y_2(x, t)$$
$$= a\sin(kx - \omega t) + a\sin(kx - \omega t + \phi)$$
$$= a\left[2\sin\left\{\frac{(kx - \omega t) + (kx - \omega t + \phi)}{2}\right\}\frac{\cos\phi}{2}\right]$$

(by using the trigonometrical relation)

$$y(x,t) = 2a\cos\frac{\phi}{2}\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

Clearly, it shows that the resultant wave is a harmonic travelling wave in positive direction of X-axis with same frequency and wavelength. The phase angle of resultant wave is $\frac{\phi}{2}$.

And the amplitude is $A(\phi) = 2a\cos\frac{\phi}{2}$

Case I For $\phi = 0$ i.e. the two waves are in phase.

$$y(x,t) = 2a\sin(kx - \omega t)$$

i.e. the resultant wave will has amplitude $2a$ (amplitude will be maximum).

Case II For $\phi = \pi$ i.e. the two waves are out of phase by 180°.

$$y(x,t) = 0$$

(i.e. zero displacement everywhere at all times)

Constructive and Destructive Interference

When waves interfere in phase, it is called **constructive interference** and when they interfere in phase opposition, that is called **destructive interference**.

Note

In generalised form For constructive interference.

- Phase difference, $\phi = 2n\pi$
- Path difference $= \Delta x = n\lambda$

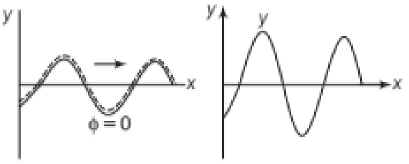
For destructive interference

- Phase difference, $\phi = (2n+1)\pi$
- Path difference, $\Delta x = (2n+1)\lambda/2$

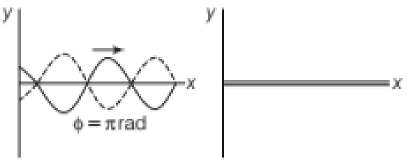
where, $n = 0, 1, 2, \ldots$

λ is the wavelength.

The phenomenon of constructive and destructive interference are shown in the figures (a) and (b), respectively.



(a) Constructive interference



(b) Destructive interference

(i) In constructive interference, if a wave meets a crest of another wave of same frequency at the same point, then the magnitude of the displacement is the sum of the individual magnitudes as shown in Fig. (a).

(ii) In destructive interference, if a crest of one wave meets a trough of another wave, then the magnitude of the displacements is equal to the difference in the individual magnitudes as shown in Fig. (b).

(iii) If two sinusoidal waves of the same amplitude and wavelength travel in the same direction along a stretched string, they interfere to produce a resultant sinusoidal wave travelling in that direction.

(iv) For interference between sources of amplitudes A_1, A_2 and associated intensities I_1, I_2 as $I \propto A^2$.

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

When $I_1 = I_2 = I_0$, then $I_{\text{resultant}} = 4I_0 \cos^2(\phi/2)$

where, ϕ is the phase difference.

(v) These principles are valid for sound and light waves as well provided sources must be coherent i.e. these phase differences must not change with passage of time.

EXAMPLE |8| Let us Mix with Each Other

Two waves of equal frequencies have their amplitudes in the ratio of 3 : 5. They are superimposed on each other. Calculate the ratio of $I_{\max}/I_{\min}$.

Sol. $\frac{A_1}{A_2} = \frac{3}{5} \Rightarrow \sqrt{\frac{I_1}{I_2}} = \frac{3}{5}$

Now, $$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right)^2$$

$$= \left(\frac{3/5+1}{3/5-1}\right)^2 = \frac{64}{4} = \frac{16}{1}$$

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Water waves produced by a motorboat sailing in water are [NCERT Exemplar]

(a) neither longitudinal nor transverse
(b) Both longitudinal and transverse
(c) only longitudinal
(d) only transverse

Sol. (b) Water waves produced by a motorboat sailing in water are both longitudinal and transverse, because the waves produced are due to the transverse as well as lateral vibrations in the particles of the medium.

2. The frequency of a sound wave is n and its velocity is v. If the frequency is increased to $4n$, the velocity of the wave will be

(a) v (b) $2v$
(c) $4v$ (d) $v/4$

Sol. (a) Velocity of sound is independent of frequency. Therefore, it is same (v) for frequency n and $4n$.

3. A steel wire has linear mass density 6.9×10^{-3} kg m^{-1}. If the wire is under a tension of 60 N, then the speed of the transverse waves on the wire is

(a) 63 ms^{-1} (b) 75 ms^{-1}
(c) 73 ms^{-1} (d) 93 ms^{-1}

Sol. (d) Linear mass density $= 6.9\times10^{-3}$ kg m^{-1}

Tension, $T = 60$ N

Thus, speed of wave on the wire is given by

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{60\ \text{N}}{6.9\times10^{-3}\ \text{kg m}^{-1}}} = 93\ \text{ms}^{-1}$$

4. The displacement of the wave given by equation $y(x,t) = a\sin(kx - \omega t + \phi)$, where $\phi = 0$ at point x and $t = 0$ is same as that at point

(a) $x + 2n\pi$ (b) $x + \frac{2n\pi}{k}$
(c) $kx + 2n\pi$ (d) Both (a) and (b)

Sol. (b) $y(x, 0) = a\sin kx = a\sin(kx + 2n\pi)$

$$= a\sin k\left(x + \frac{2n\pi}{k}\right)$$

$\Rightarrow$ The displacement at points x and $\left(x + \frac{2n\pi}{k}\right)$ are the same where, $n = 1, 2, 3, \ldots$

5. Two pulses having equal and opposite displacements moving in opposite directions overlap at $t = t_1$s. The resultant displacement of the wave at $t = t_1$s is

(a) twice the displacement of each pulse
(b) half the displacement of each pulse
(c) zero
(d) Either (a) or (c)

Sol. (c) The displacement due to two pulses will exactly cancel out each other. Thus, there will be no displacement throughout.

VERY SHORT ANSWER Type Questions

VERY SHORT ANSWER Type Questions

6. What is the quantity transmitted with propagation of longitudinal waves through a medium? **[NCERT Exemplar]**

Sol. Propagation of longitudinal waves through a medium leads to transmission of energy through the medium.

7. Is Newton's law of motion applicable for material waves? Is this applicable for electromagnetic waves?

Sol. Newton's laws of motion are applicable for material waves but not applicable for electromagnetic waves.

8. Does a vibrating source always produce sound?

Sol. A vibrating source produces sound when it vibrates in a medium and frequency of vibration lies within the audible range (20 Hz to 20 kHz).

9. How speed of sound waves in air varies with humidity? **[NCERT Exemplar]**

Sol. Speed of sound waves in air increases with increase in humidity. This is because, presence of moisture decreases the density of air.

10. The displacement of an elastic wave is given by the function $y = 3\sin\omega t + 4\cos\omega t$

where, y is in cm and t is in second. Calculate the resultant amplitude. **[NCERT Exemplar]**

Sol. The resultant amplitude will be

$$y = \sqrt{y_1^2 + y_2^2} = \sqrt{9+16} = \sqrt{25} = 5\,\text{cm}$$

11. Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where, its speed is $2v$ m/s. Wavelength of sound waves in the second medium is **[NCERT Exemplar]**

Sol. Frequency in the first medium, $\nu = \dfrac{v}{\lambda}$

Frequency will remain same in the second medium, as the source.

$$\Rightarrow \quad \nu' = \nu \Rightarrow \frac{2v}{\lambda'} = \frac{v}{\lambda} \Rightarrow \lambda' = 2\lambda$$

SHORT ANSWER Type Questions

12. A steel wire has a length of 12 m and a mass of 2.10 kg. What will be the speed of a transverse wave on this wire when a tension of 2.06×10^4N is applied? **[NCERT Exemplar]**

Sol. $l = 12\,\text{m}$, $M = 2.10$ kg, $T = 2.06 \times 10^4$ N, $v = ?$

$$\mu = \frac{M}{l} = \frac{2.10}{12}\ \text{kg/m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{2.06 \times 10^4}{2.10/12}}$$

$$= 3.43 \times 10^2\ \text{m/s}$$

13. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at $20°\text{C} = 343\ \text{ms}^{-1}$? **[NCERT]**

Sol. Given, $l = 12.0$ m, $M = 2.10$ kg, $T = ?$, $v = 343$ m/s

μ = mass per unit length

$$= \frac{M}{l} = \frac{2.10}{12} = 0.175\ \text{kgm}^{-1}$$

As we know that, $v = \sqrt{\dfrac{T}{\mu}}$

$$\Rightarrow \quad T = v^2\mu = (343)^2 \times (0.175)$$

$$\Rightarrow \quad T = 2.06 \times 10^4\ \text{N}$$

14. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in **[NCERT Exemplar]**

Sol. Here, $\mu = \dfrac{2.5}{20}$ kg/m, $T = 200$ N

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{2.5/20}} = \sqrt{\frac{200 \times 20}{2.5}}$$

$$= \sqrt{\frac{4 \times 10^4}{25}} = \frac{2 \times 10^2}{5}$$

$$= \frac{20 \times 10}{5} = 40\ \text{m/s}$$

So, $\quad t = \dfrac{l}{v} = \dfrac{20}{40} = 0.50$ s

15. Equation of a plane progressive wave is given by $y = 0.6\sin 2\pi\left(t - \dfrac{x}{2}\right)$. On reflection from a denser medium, its amplitude becomes 2/3 of the amplitude of incident wave. What will be equation of reflected wave? **[NCERT Exemplar]**

Sol. On reflection from the denser medium, there will be a phase change of 180°.

Net amplitude $= \frac{2}{3} \times 0.6 = 0.4$

Hence, equation of reflected wave will be

$$y = 0.4 \sin 2\pi\left[t + \frac{x}{2} + \pi\right]$$
$$= -0.4 \sin 2\pi(t + x/2)$$

16. At what temperature (in °C) will the speed of sound in air be 3 times its value at 0°C? **[NCERT Exemplar]**

Sol. We know that, speed, $v \propto \sqrt{T}$

By formula $v = \frac{\chi RT}{\rho}$

where, T is in kelvin.

$$\frac{v_t}{v_0} = \sqrt{\frac{273+t}{273+0}} = 3$$

$\Rightarrow \quad \frac{273+t}{273} = 9$

$\Rightarrow \quad t = 9 \times 273 - 273$
$= 2184°C$

17. You have learnt that a travelling wave in one dimension is represented by a function $y = f(x,t)$ where, x and t must appear in the combination $x - vt$ or $x + vt$, i.e. $y = f(x \pm vt)$. Is the converse true? Examine if the following functions for y can possibly represent a travelling wave

(i) $(x - vt)^2$
(ii) $\log[(x + vt)/x_0]$
(iii) $1/(x + vt)$ **[NCERT Exemplar]**

Sol. Conceptual question based on fundamentals of characteristics of a travelling wave.

The converse is not true means if the function can be represented in the form $y = f(x \pm vt)$, it does not necessarily express a travelling wave. As the essential condition for a travelling wave is that the vibrating particle must have finite displacement value for all x and t.

(i) For $x = 0$,

If $t \to 0$, then $(x - vt)^2 \to 0$ which is finite, hence, it is a wave as it passes the two tests.

(ii) $\log\left(\frac{x + vt}{x_0}\right)$

At $x = 0$ and $t = 0$,

$$f(x,t) = \log\left(\frac{0+0}{x_0}\right)$$
$$= \log 0 \to \text{not defined}$$

Hence, it is not a wave.

(iii) $\frac{1}{x + vt}$

For $x = 0, t = 0, f(x) \to \infty$

Though the function is of $(x \pm vt)$ type still at $x = 0$, it is infinite, hence, it is not a wave.

LONG ANSWER Type I Questions

18. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 kms^{-1} and that of P wave is 8.0 kms^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur? **[NCERT]**

Sol. Let v_1, v_2 be the velocities of S wave and P wave and t_1, t_2 be the time taken by these waves to reach the seismograph.

l = distance of occurrence of earthquake from the seismograph

$$v_1 t_1 = v_2 t_2$$

$\Rightarrow \quad v_1 = 4 \text{ km s}^{-1}, v_2 = 8 \text{ km s}^{-1}$

$\Rightarrow \quad 4t_1 = 8t_2 \Rightarrow t_1 = 2t_2 \quad$ ...(i)

$t_1 - t_2 = 4 \text{ min} = 240\text{s} \quad$ ...(ii)

On solving Eqs. (i) and (ii), $t_2 = 240\text{s}$

$\Rightarrow \quad t_1 = 2t_2 = 2 \times 240 = 480\text{s}$

$\Rightarrow \quad l = v_1 t_1 = 4 \times 480 = 1920 \text{km}$

19. A stone dropped from the top of a tower of height 300 m in high splashes into the water of a pond near the base of the tower. When is the splash heard at the top, given that the speed of sound in air is 340 ms^{-1}? (Take, $g = 9.8 \text{ ms}^{-2}$) **[NCERT]**

Sol. Given, $h = 300$ m, $g = 9.8$ m/s^2, $v = 340$ ms^{-1}

t_1 = time taken by stone to strike the water surface

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{300}{4.9}} = 7.82 \text{ s} \quad \left(\text{As } h = 0 + \frac{1}{2}gt_1^2\right)$$

t_2 = time taken by the splash's sound to reach top of the tower

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.882 \quad \left[\because v = \frac{h}{t_2}\right]$$

Total time, t = time to hear splash of sound
$= t_1 + t_2 = 7.82 + 0.882 = 8.702$

20. If c is rms speed of molecules in a gas and v is the speed of sound waves in the gas, show that c/v is constant and independent of temperature for all diatomic gases. **[NCERT Exemplar]**

Sol. From kinetic theory of gases,

$p = \frac{1}{3}\rho c^2$, where c is rms speed of molecules of gas.

$$\Rightarrow \qquad c = \sqrt{\frac{3p}{\rho}} \qquad \text{...(i)}$$

$$v = \text{speed of sound in the gas} = \sqrt{\frac{\gamma p}{\rho}} \qquad \text{...(ii)}$$

$\Rightarrow$ from Eqs. (i) and (ii),

$$\frac{c}{v} = \sqrt{\frac{3p}{\rho} \times \frac{\rho}{\gamma p}} = \sqrt{\frac{3}{\gamma}}$$

For diatomic gases, $\gamma = 1.4$ = constant

$$\Rightarrow \qquad \frac{c}{v} = \sqrt{\frac{3}{1.4}} = 1.46 = \text{constant}]$$

21. One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at the end have negligible amplitude. At $t = 0$, the left and (fork end) of the string $x = 0$ has zero transverse displacement ($y = 0$) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string. **[NCERT]**

Sol. $\nu = 256$ Hz, $T = m \times g$, $T = 90 \times 9.8 = 882$ N

$$\mu = \frac{m}{L} = 8.0 \times 10^{-3} \text{kgm}^{-1}$$

Amplitude, $a = 5$ cm $= 0.05$ m

Velocity of the transverse wave

$$\Rightarrow \qquad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{882}{8 \times 10^{-3}}} = 3.32 \times 10^2 \text{ m/s}$$

$$\omega = 2\pi\nu = 2 \times 3.14 \times 256$$

$$= 1.61 \times 10^3 \text{ rad/s}$$

$$\lambda = \frac{v}{\nu} = \frac{3.32 \times 10^2}{256}$$

$$k = \frac{2\pi}{\lambda} = \frac{2 \times 3.14 \times 256}{3.32 \times 10^2} = 4.84 \text{ m}^{-1}$$

As wave propagating along positive X-axis

$$Y = a\sin(\omega t - kx)$$

$$= 0.05\sin(1.61 \times 10^3 t - 4.84x)$$

Here x, y are in metre and t is in second.

22. For the wave $y(x,t) = 3.0\sin(36t + 0.018x + \pi/4)$, plot the displacement (y) *versus* (t) graph for $x = 0$, 2 and 4 cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another amplitude, frequency or phase? **[NCERT]**

Sol. The transverse harmonic wave is

$$y(x,t) = 3.0\sin\left[36t + 0.018x + \frac{\pi}{4}\right]$$

It is the equation of travelling wave along negative direction of x.

For $\qquad x = 0, \qquad$ ...(i)

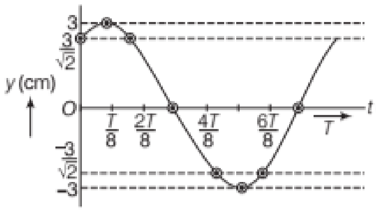
$$y(x,t) = 3.0\sin[36t + \pi/4]$$

Here, $\qquad \omega = \frac{2\pi}{T} = 36, \quad T = \frac{2\pi}{36} = \frac{\pi}{18}$ s

For different values of t, we calculate y using Eq. (i). These values are tabulated below

t	0	$T/8$	$2T/8$	$3T/8$	$4T/8$	$5T/8$	$6T/8$	$7T/8$	T
y	$3/\sqrt{2}$	3	$3/\sqrt{2}$	0	$-3/\sqrt{2}$	-3	$-3/\sqrt{2}$	0	$3/\sqrt{2}$

On plotting y *versus* t graph, we obtain a sinusoidal curve as shown in figure below.



Similar graphs are obtained for $x = 2$ cm and $x = 4$ cm. The oscillatory motion in travelling wave differs from one point to another only in terms of phase. Amplitude and frequency of oscillatory motion remain the same in all the three cases.

23. A transverse harmonic wave on a string is described by $y(x, t) = 3.0\sin(36t + 0.018x + \pi/4)$ Where, x and y are in cm and t in seconds. The positive direction of x is from left to right.

(i) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

(ii) What are its amplitude and frequency?

(iii) What is the initial phase at the origin?

(iv) What is the least distance between two successive crests in the wave? **[NCERT]**

Sol. Given equation is $y(x, t) = 3.0\sin(36t + 0.018x + \pi/4)$

Comparing with standard equation

$$y(x, t) = a\sin(\omega t + kx + \phi)$$

(i) The given equation represents a transverse harmonic wave travelling from right to left (i.e. along negative x-axis). It is not a stationary wave.

By comparing, we get $\omega = 36$ rad/s, $k = 0.018$/cm

$\therefore$ Speed of wave, $v = \dfrac{\omega}{k} = \dfrac{36}{0.018} = 2000$ cm/s

(ii) By comparing amplitude, $a = 3$ cm

$$\Rightarrow \quad 2\pi v = 36$$

$$\Rightarrow \quad v = \frac{36}{2\pi} = 5.73 \text{ Hz}$$

(iii) Initial phase, $\phi = \dfrac{\pi}{4}$

(iv) $\omega = 36$, $k = \dfrac{2\pi}{\lambda} = 0.018$

$$\Rightarrow \lambda = \text{least distance} = \frac{2\pi}{k} = \frac{2\pi}{0.018} \text{ cm}$$

$$= 349.1 \text{ cm}$$

24. A source of frequency 250Hz produces sound waves of wavelength 1.32 m in a gas at STP. Calculate the change in the wavelength, when temperature of the gas is 40°C

Sol. We have, $v_0 = 250$Hz, $T_0 = 273$K

$$T_1 = 273 + 40 = 313\text{K}; \ \lambda_0 = 1.32\text{m}$$

$\therefore$ Speed of sound, $v_0 = v_0 \cdot \lambda_0 = 250 \times 1.32 = 330$ m/s

As we know that,

Speed of sound, $v \propto \sqrt{T}$

Thus, $$\frac{v_1}{v_0} = \sqrt{\frac{T_1}{T_0}}$$

$$v_1 = v_0\sqrt{\frac{T_1}{T_0}} = 330\sqrt{\frac{313}{273}} = 353.34 \text{ m/s}$$

$\because$ $$v_1 = v_0 \lambda_1$$

$$\lambda_1 = \frac{353.34}{250} = 1.41 \text{ m}$$

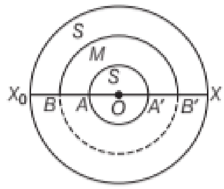
$\therefore$ Change in the wavelength,

$$\Delta\lambda = \lambda_1 - \lambda_0 = 1.41 - 1.32 = 0.09 \text{ m}$$

LONG ANSWER Type II Questions

25. The earth has a radius of 6400 km. The inner core of 1000 km radius is solid. Outside it, there is a region from 1000 km to a radius of 3500 km which is in molten state. Then, again from 3500 km to 6400 km the earth is solid. Only longitudinal (P) waves can travel inside a liquid. Assume that the P wave has a speed of 8 kms^{-1} in solid parts and of 5 kms^{-1} in liquid parts of the earth. An earthquake occurs at some place close to the surface of the earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter? **[NCERT Exemplar]**

Sol. Consider the diagram shown below



The wave is travelling along the diameter X_0X.

(i) x_1 = distance through solid portion

$$= X_0B + AA' + B'X$$

$$= (2900 + 2 \times 1000 + 2900) \text{ km} = 7800 \text{ km}$$

(ii) x_2 = distance through molten portion

$$= BA + A'B' = (2500 + 2500) \text{ km} = 5000 \text{ km}$$

Now, $v_1 = 8$ km/s, $v_2 = 5$ km/s, t = total time

$$\left[\text{Time} = \frac{\text{Distance}}{\text{Speed}} \ t = \frac{x}{v}\right]$$

$$= t_1 + t_2 = \frac{x_1}{v_1} + \frac{x_2}{v_2} = \frac{7800}{8} + \frac{5000}{5}$$

$$t = 975 + 1000 = 1975 \text{ s} = 32 \text{ minutes } 55 \text{ seconds}$$

26. In the given progressive wave,

$$y = 5\sin(100\pi t - 0.4\pi x)$$

where, y and x are in m, t is in seconds. What is the

(i) amplitude, (ii) wavelength,

(iii) frequency, (iv) wave velocity,

(v) particle velocity amplitude? **[NCERT Exemplar]**

Sol. Comparing with the standard form of equation

$$y = a\sin[\omega t - kx]$$

(i) Amplitude, $a = 5$ m

(ii) $\omega = \dfrac{2\pi}{T} = 100\pi$

$$k = \frac{2\pi}{\lambda} = 0.4\pi \Rightarrow \lambda = \frac{2}{0.4} = \frac{20}{4} = 5 \text{ m}$$

(iii) $\omega = 2\pi v = 100\pi \Rightarrow v = 50$ Hz

(iv) Wave velocity, $v = v\lambda = 50 \times 5 = 250$ m/s

(v) Particle velocity $= \dfrac{dy}{dt} = a\omega\cos(\omega t - kx)$

$$\left(\frac{dy}{dt}\right)_{max} = a\omega = 5 \times 100\pi = 500\pi \text{ m/s}$$

27. Use the formula, $v = \sqrt{\dfrac{\gamma P}{\rho}}$ to explain, why the speed of sound in air

(i) is independent on pressure,

(ii) increases with temperature,

(iii) increases with humidity. **[NCERT]**

Sol. (i) Effect of pressure

v = speed of sound in a gas $= \sqrt{\frac{\gamma p}{\rho}}$

p = pressure, ρ = density, $\frac{M}{V} \Rightarrow v = \sqrt{\frac{\gamma pV}{M}}$

When T is constant, pV = constant $\Rightarrow v$ = constant

Hence, velocity of sound is independent of the change in pressure of the gas provided temperature remains constant.

(ii) Formula for velocity, $v = \sqrt{\frac{\gamma p}{\rho}}$

According to standard gas equation,

$$pV = RT$$

$$\Rightarrow \quad p = \frac{RT}{V}$$

$$\Rightarrow \quad v = \sqrt{\frac{\gamma \times RT}{pV}} = \sqrt{\frac{\gamma RT}{M}}$$

Where, $M = pV$ = molecular weight of the gas

$$\Rightarrow \quad v \propto \sqrt{T}$$

Hence, v increases with temperature.

(iii) Due to presence of water vapours in air density changes. Hence, velocity of sound changes with humidity.

Let ρ_m = density of moist, ρ_d = density of dry air

v_m = velocity of sound in moist air

v_d = velocity of sound in dry air

$$v_m = \sqrt{\frac{\gamma p}{\rho_m}}, \; v_d = \sqrt{\frac{\gamma p}{\rho_d}}$$

$$\frac{v_m}{v_d} = \sqrt{\frac{\rho_d}{\rho_m}} \text{ as } \rho_d > \rho_m \Rightarrow v_m > v_d$$

28. For the harmonic travelling wave $y = 2\cos 2\pi(10t - 0.0080x + 3.5)$ where, x and y are in cm and t is in second. What is the phase difference between the oscillatory motion at two points separated by a distance of

(i) 4 m (ii) 0.5 m (iii) $\frac{\lambda}{2}$

(iv) $\frac{3\lambda}{4}$ (at a given instant in time)?

(v) What is the phase difference between the oscillation of a particle located at $x = 100$ cm, at $t = T$ s and $t = 5$ s? **[NCERT Exemplar]**

Sol. Given equation is $y = 2\cos 2\pi(10t - 0.008x + 3.5)$

Comparing with standard equation,

$$y = a\cos(\omega t - kx + \phi)$$

$$a = 2 \text{ cm}, \omega = \frac{2\pi}{T} = 20\pi, \; T = 0.1 \text{ s}$$

$$k = \frac{2\pi}{\lambda} = 0.008 \times 2\pi$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{2\pi \times 0.008} = 1.25 \text{ m}$$

$$\phi = 2\pi \times 3.5 = 7\pi \text{ rad}$$

(i) $\phi_1 = \frac{2\pi}{\lambda} \times 4 = \frac{2\pi}{1.25} \times 4 = 6.4\pi$ rad

(ii) $\phi_2 = \frac{2\pi}{\lambda} \times x = \frac{2\pi}{1.25} \times 0.5 = 0.8\pi$ rad

(iii) When $x = \lambda/2$

$$\phi_3 = \frac{2\pi}{\lambda} \times \lambda/2 = \pi \text{ rad}$$

(iv) When $x = \frac{3\lambda}{4}$; $\phi_4 = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}$ rad

(v) At $t = T$; $\phi = \frac{2\pi}{T} = \frac{2\pi}{0.1} = 20\pi$ rad

and at $t = 5$s; $\phi' = \frac{2\pi}{0.1} \times 5.100\pi$ rad

$\therefore$ Phase difference $\phi' - \phi = 100\pi - 20\pi = 80\pi$ rad

29. A travelling harmonic wave on a string is described by $y(x,t) = 7.5\sin(0.0050x + 12t + \pi/4)$.

(i) What are the displacement and velocity of oscillation of a point at $x = 1$ cm, and $t = 1$ s? Is this velocity equal to the velocity of wave propagation?

(ii) Locate the points of the string which have the same transverse displacements and velocity as the $x = 1$ cm point at $t = 2$ s, 5 s and 11 s. **[NCERT]**

Sol. The travelling harmonic wave is

$$y(x, t) = 7.5\sin(0.0050x + 12t + \pi/4)$$

At $x = 1$ cm and $t = 1$ s

$$y(1, 1) = 7.5\sin(0.005 \times 1 + 12 \times 1 + \pi/4)$$

$$= 7.5\sin(12.005 + \pi/4) \quad \text{...(i)}$$

Now, $\theta = (12.005 + \pi/4)$ rad

$$= \frac{180}{\pi}(12.005 + \pi/4) \text{ degree}$$

$$= \frac{12.79 \times 180}{22/7} = 732.55°$$

$\therefore$ From Eq. (i), $y(1, 1) = 7.5\sin(732.55°)$

$$= 7.5\sin(720 + 12.55°)$$

$$= 7.5\sin 12.55° \text{ cm}$$

$$= 7.5 \times 0.2173 = 1.63 \text{ cm}$$

Velocity of oscillation,

$$v = \frac{d}{dt}\{y(1, 1)\}$$

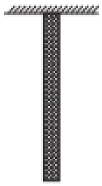
$$= \frac{d}{dt}\left[7.5\sin\left(0.005x + 12t + \frac{\pi}{4}\right)\right]$$

$$= 7.5 \times 12\cos\left[0.005x + 12t + \frac{\pi}{4}\right]$$

(i) At, $x = 1$ cm, $t = 1$ s

$$v = 7.5 \times 12 \cos (0.005 + 12 + \pi/4)$$
$$= 90 \cos (732.55°) = 90 \cos (720 + 12.55)$$
$$v = 90 \cos (12.55°) = 90 \times 0.9765 = 87.89 \text{ cm/s}$$

Comparing the given equations with the standard form



$\Rightarrow y(x, t) = a \sin(kx + \omega t + \phi)$

We get, $a = 7.5$ cm, $\omega = 12$, $2\pi\nu = 12$ or $\nu = \frac{6}{\pi}$

$$\frac{2\pi}{\lambda} = 0.005$$

$$\therefore \lambda = \frac{2\pi}{0.005} = \frac{2 \times 3.14}{0.005} = 1256 \text{ cm} = 12.56 \text{ m}$$

Velocity of wave propagation,

$$v = \nu\lambda = \frac{6}{\pi} \times 12.56 \text{ m/s} = 24 \text{ m/s}$$

We find that velocity at $x = 1$ cm, $t = 1$ s is **not equal to velocity of wave propagation.**

(ii) Now, all points which are at a distance of $\pm\lambda, \pm 2\lambda, \pm 3\lambda$ from $x = 1$ cm will have same transverse displacement and velocity. As $\lambda = 12.56$ m, therefore, all points at distances ± 12.6 m, ± 25.2 m, ± 37.8 m from $x = 1$ cm will have same displacement and velocity, as $x = 1$ cm point at $t = 2$s, 5 s and 11 s.

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. The wave generated from up and down jerk given to the string or by up and down motion of the piston at end of the pipe is

(a) transverse (b) longitudinal

(c) Both (a) and (b) (d) electromagnetic wave

2. With propagation of longitudinal waves through a medium, the quantity transmitted is

(a) matter **[NCERT Exemplar]**

(b) energy

(c) energy and matter

(d) energy, matter and momentum

3. Speed of sound wave in air **[NCERT Exemplar]**

(a) is undependent of temperature

(b) increases with pressure

(c) increases with increase in humidity

(d) decreases with increase in humidity

4. Equation of progressive wave is

$$y = a \sin\left(10\pi x + 11\pi t + \frac{\pi}{3}\right)$$

The wavelength of the wave is

(a) 0.2 unit (b) 0.1 unit (c) 0.5 unit (d) 1 unit

5. Two sine waves travel in the same direction in a medium. The amplitude of each wave is A and the phase difference between the two waves is 120°. The resultant amplitude will be

(a) A (b) $2A$ (c) $4A$ (d) $\sqrt{2}A$

Answer

1. (c)	2. (b)	3. (c)	4. (a)	5. (a)

VERY SHORT ANSWER Type Questions

6. A wave pulse is described by $y(x,t) = ae^{-(bx-at)^2}$ where a, b and c are positive constants. What is the speed of this wave? **[Ans.** c/b**]**

7. The density of oxygen is 16 times the density of hydrogen. What is the relation between the speed of sound in two gases? **[Ans.** $V_{H_2} = 4\ V_{O_2}$**]**

8. The velocity of sound in a tube containing air at 27°C and a pressure of 76 cm of mercury is 330 m/s. What will it be when pressure is increased to 100 cm of mercury and the temperature is kept constant? **[Ans.** will remain same**]**

9. The ratio of the amplitudes of two waves are 3 : 4. What is the ratio of the intensities of two waves? **[Ans.** 9 : 16**]**

SHORT ANSWER Type Questions

|2 Marks|

10. Write expression for the intensity related with a string through which a wave is passing.

11. If the speed of a transverse wave on a stretched string of length 1 m is 60 m/s. What is the fundamental frequency of vibration? **[Ans.** 30 Hz**]**

12. The equation of a wave travelling on a string stretched along the x-axis is given by

$$Y = ke^{-\left(\frac{x}{b}+\frac{t}{T}\right)^2}$$

where, is the maximum of the pulse located at $t = T$? At $t = 2T$? **[Ans.** $x = -b$ and $x = -2b$**]**

LONG ANSWER Type I Questions

13. Two identical sinusoidal waves travel along a stretched string in the positive direction of x-axis. Show graphically resultant wave formed due to their superposition when phase difference between them is (i) 0 (ii) π.

14. Write condition for constructive and destructive interferences.

15. A certain 120 Hz wave on a string has an amplitude of 0.160 mm. How much energy exists in an 80 g length of the string? [**Ans.** 0.58 mJ]

16. A travelling wave pulse is given by $y = \dfrac{10}{5+(x+2t)^2}$

Here, x and y are in metre and t in second. In which direction and with what velocity is the pulse propagating? What is the amplitude of pulse?
[**Ans.** velocity = 2 m / s; amplitude = 2 m; along negative X - axis]

LONG ANSWER Type II Questions

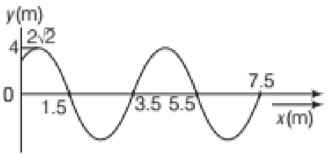
17. A transverse wave of amplitude 0.5 mm and frequency 100 Hz is produced on a wire stretched to a tension of 100 N. If the wave speed is 100 m/s. What average power is the source transmitting to the wire? [**Ans.** 49 mm]

18. Discuss Newton's formula for the velocity of longitudinal waves in air. What correction was applied by Laplace and why?

19. Find condition for constructive interference of sound wave in terms of phase difference.

20. A wire of variable mass per unit length $\mu = \mu_0 y$ is hanging from the ceiling as shown in figure. The length of wire is l_0. A small transverse disturbance is produced at its lower end. Find the time after which the disturbance will reach to the other end.
$\left[\textbf{Ans. } \sqrt{\dfrac{8l_0}{g}}\right]$

21. The figure shows a snap photograph of a vibrating string at $t = 0$. The particle P is observed moving up with velocity $20\sqrt{3}$ cm/s. The tangent at P makes an angle 60° with x-axis



(i) Find the direction in which the wave is moving.
(ii) Write the equation of the wave.
(iii) The total energy carried by the wave per cycle of the string. Assuming that the mass per unit length = 50 g/m of the string.

[**Ans.** (i) Negative x. (ii) $Y = (0.4\text{ cm})\sin\left(10\pi t + \dfrac{\pi}{2}x + \dfrac{\pi}{4}\right)$, (iii) 1.6×10^{-5} J]

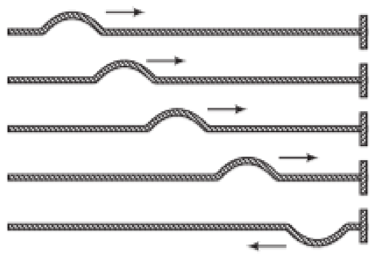
|TOPIC 2|
Stationary Waves and Doppler's Effect

REFLECTION OF WAVES

If a pulse or wave meets a rigid boundary, they get reflected e.g. echo. If the boundary is not completely rigid, a part of the incident wave is reflected and a part is transmitted into the second medium.

If a wave is incident obliquely on the boundary between two different media, the transmitted wave is called the **refracted wave**. The incident and refracted waves obey Snell's law of refraction and incident and reflected waves obey usual laws of reflection.

Reflection of a pulse meeting a rigid boundary is shown in the figure below



Reflection of pulse meeting a rigid boundary

In the above diagram, the travelling pulse is moving towards the non-absorptive wall (i.e. there is no absorption of energy at the boundary). As the displacement of connecting point at the boundary must be zero, hence by the principle of superposition, the reflected wave must have equal amplitude as well as phase difference of π.

If boundary point is completely free to move, the reflected pulse has the same phase and amplitude as the incident pulse. The net maximum displacement at the boundary will be twice of amplitude of each pulse.

Let the incident travelling wave is

$$y(x, t) = a \sin(kx - \omega t)$$

At the rigid boundary, the reflected wave is given by

$$y_r(x, t) = a \sin(kx - \omega t + \pi)$$
$$= -a \sin(kx - \omega t)$$

At an open boundary, the reflected wave is given by

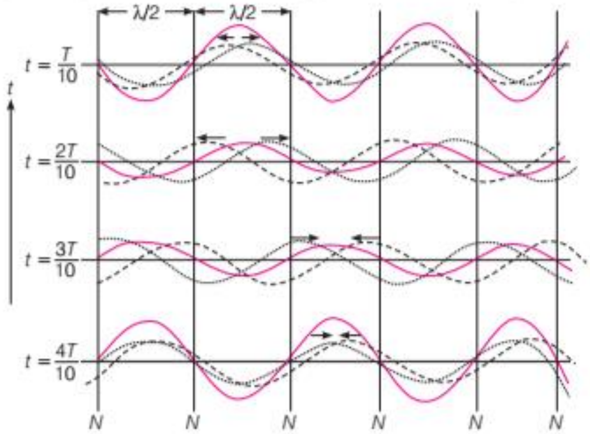
$$y_r(x, t) = a \sin(kx - \omega t + 0)$$
$$= a \sin(kx - \omega t)$$

At the rigid boundary, $y' = y + y_r = 0$ at all times.

STANDING WAVES AND NORMAL MODES

Standing Waves

A new set of waves formed when two sets of progressive wave trains of the same type, i.e. both transverse or both longitudinal having the same amplitude and same time period/frequency/wavelength and travelling with same speed along the same straight line in opposite directions superimpose are called **standing waves** or **stationary waves**.



Stationary waves arising from superposition of two harmonic waves travelling in opposite directions

The resultant waves do not propagate in any direction, there is no transfer of energy in the medium.

When we are considering reflections at two or more boundaries (a string fixed at both ends or an air column in a pipe with both ends closed) here due to interference between incident and reflected wave, a steady wave pattern will set up on the string. Such wave patterns are called **standing waves** or **stationary waves.**

Now, consider wave equations

$y_1(x, t) = a \sin(kx - \omega t)$ (for wave along +ve X-axis)

$y_2(x, t) = a \sin(kx + \omega t)$ (for wave along −ve X-axis)

Resultant wave, $y(x, t) = y_1(x, t) + y_2(x, t)$

$$= a[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

$$\boxed{\text{Net resultant wave, } y(t) = 2a \sin kx \cos \omega t}$$

Here, the amplitude of the standing/stationary wave is $2a \sin kx$.

Thus, in this wave pattern, the amplitude varies from point to point, but each element of the string oscillates with the same angular frequency ω or time period and there is no phase difference.

As the wave pattern remains stationary, the amplitude is fixed at a given location but different at different locations.

Normal Modes

Points having zero amplitude (i.e. where, there is no motion are called **Nodes** and the points having largest amplitude are known as **Antinodes**.

The significant feature of stationary waves is that the system cannot oscillate with any arbitrary frequency, but is characterised by a set of natural frequencies or **normal modes** of oscillation.

Now, let us determine these normal modes for a stretched string fixed at both ends.

From above equation for **nodes,** the amplitude is zero for values of kx that give $\sin kx = 0$

$\sin kx = 0 \Rightarrow kx = n\pi; n = 0, 1, 2, 3, 4, \ldots$

$$\Rightarrow \boxed{\text{For nodes, } x = \frac{n\pi}{k}}$$

$$x = \frac{n\pi}{2\pi/\lambda} = \frac{n\lambda}{2}; n = 0, 1, 2, 3 \ldots$$

So, the first node is formed at $x = 0$. The second node is formed at $x = \frac{\lambda}{2}$ and so on.

Hence, distance between two successive nodes is $\frac{\lambda}{2}$.

For **antinodes,** the maximum value of amplitude is $2a$ which occurs for values of kx that give $|\sin kx| = 1$

$$\Rightarrow \quad kx = \left(n + \frac{1}{2}\right)\pi \,;\, n = 0, 1, 2, 3 \ldots$$

$$\Rightarrow \quad \boxed{\text{For antinodes, } x = (2n+1)\frac{\lambda}{4}}$$

$$n = 0, 1, 2, \ldots \left(\because k = \frac{2\pi}{\lambda}\right) \quad \ldots\text{(i)}$$

Thus, the amplitude of wave will be maximum at the position of string given by above equation. The first maximum amplitude will be at $x = \frac{\lambda}{4}$. The second maximum amplitude will be at $x = \frac{3\lambda}{4}$ and so on.

Hence, distance between two consecutive antinodes is $\frac{\lambda}{2}$.

EXAMPLE |1| Wave function

The two individual wave functions are

$y_1 = (5 \text{ cm}) \sin (4x - t)$ and $y_2 = (5 \text{ cm}) \sin (4x + t)$

where, x and y are in centimeters. Find out the maximum displacement of the motion at $x = 2.0$ cm. Also, find the positions of nodes and anti-nodes.

As from the given equations, it is clear that these two waves are travelling in opposite direction, i.e. one is moving in $+x$-direction and other in $-x$-direction. If these two waves are summed, the resulting wave will be a stationary wave.

Sol. Given, $y_1 = (5 \text{ cm}) \sin (4x - t)$

$y_2 = (5 \text{ cm}) \sin (4x + t)$

The resulting wave

$$y = (2A \sin kx) \cos \omega t$$

Now, compare the given equation.

$y_1 = (5 \text{ cm}) \sin (4x - t)$ with $y_1 = A \sin (kx - \omega t)$.

$A = 5$ cm, $k = 4$ and $\omega = 1$ rad/s

$$y = (2A \sin kx) \cos \omega t$$

$$y = (10 \sin 4x) \cos t$$

The maximum displacement of the motion at position

$x = 2{\cdot}0$ cm

$$y_{max} = 10 \sin 4x \,|_{x = 2.0}$$

$$= 10 \sin (4 \times 2) = 10 \sin (8 \text{ rad})$$

$$y_{max} = 9.89 \text{ cm}$$

The wavelength by using the relation between wavelength and wave number.

$$k = \frac{2\pi}{\lambda} = 4$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{4} = \frac{\pi}{2} \text{ cm}$$

The nodes and anti-nodes can be given as

Nodes at $x = \frac{n\lambda}{2} = n \times \left(\frac{\pi}{4}\right)$ cm,

where $n = 0, 1, 2, \ldots$

Antinodes at $x = (2n+1)\frac{\lambda}{4} = (2n+1) \times \left(\frac{\pi}{8}\right)$ cm,

where $n = 0, 1, 2, \ldots$

Vibration of String Fixed at Both Ends

Consider a stretched string having length L fixed at both ends. The one end be at $x = 0$ fixed while the other one at $x = L$. These are the boundary conditions.

As, $\quad L = \frac{n\lambda}{2} \,;\, n = 1, 2, 3 \ldots$

$\Rightarrow$ Possible wavelengths of stationary waves are

$$\lambda = \frac{2L}{n} \,;\, n = 1, 2, 3 \ldots$$

and corresponding frequencies are

Frequency of the vibration, $\nu = \frac{nv}{2L}$ for $n = 1, 2, 3 \ldots$

Here, v is the speed of the wave.

Modes of Vibration

The manner in which the string vibrates and give rise to a standing wave is known as **mode of vibration** of the string.

First Mode of Vibration

The mode of vibration in which string will vibrate in one segment or the lowest possible natural frequency (as shown in figure) is called **fundamental mode** or **first harmonic.**

As, $L = \frac{n\lambda}{2}$ for first harmonic, put $n = 1$ then,

$$L = \frac{\lambda_1}{2} \quad \text{and} \quad \nu = \frac{v}{2L}$$

Second Mode of Vibration

The mode of vibration in which string will vibrate in two segments (as shown in figure) is called **second harmonic** or **first overtone.** For the second harmonic, put $n = 2$ in $L = \frac{n\lambda}{2}$

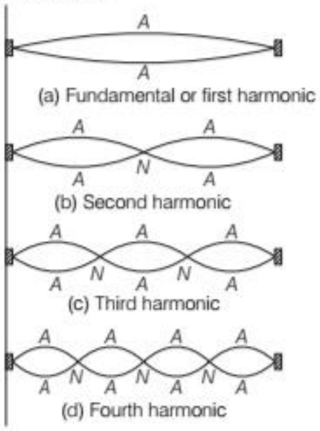
For this, $L = \lambda_2$ and $\nu_2 = \frac{2v}{2L}$ or $2\nu_1$

As, $\quad \nu = \frac{nv}{2L}$

Third Mode of Vibration

If $n = 3$, the frequency, $\nu = \frac{3v}{2L}$ or $\nu = 3\nu_1$ is called **third harmonic** and so on.

Figure below shows the first four harmonics of a stretched string fixed at both ends.



The vibration of a string will be a superposition of different modes. Some modes may be strongly excited and some less.

Note Musical instruments like sitar or violin are based on this principle. Where the string is plucked or bowed, determines which modes are more prominent than others.

EXAMPLE |2| Let's Play the Guitar
A guitar string is 100 cm long and has a fundamental frequency of 125 Hz. Where should it be pressed to produce a fundamental frequency of 200 Hz ?

Sol. Fundamental frequency of a string fixed at both end is

$$\nu = \frac{v}{2L}$$

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad \left[\because v = \sqrt{\frac{T}{\mu}}\right]$$

As T and μ are fixed

$$\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1} \Rightarrow L_2 = \frac{\nu_1}{\nu_2} L_1$$

$$\Rightarrow \qquad L_2 = \frac{125\text{ Hz}}{200\text{ Hz}} \times 100 = \frac{125}{2} = 62.5\text{ cm}$$

Vibration of a String Fixed at one End

Standing waves can also be produced on a string which is fixed at one end and whose other end is free to move in a transverse direction. Here, in this case, we will have antinode at the free end and node at the fixed end.

Now, consider the equation of standing wave

$$y(x, t) = 2a \sin kx \cos \omega t$$

As we are having antinode at the end, $x = L$

$$\Rightarrow \qquad \sin kL = \pm 1$$

$$kL = (2n+1)\frac{\pi}{2}, \text{ where } n = 0, 1, 2, \dots$$

$$\Rightarrow \qquad \frac{2\pi}{\lambda} \times L = (2n+1)\frac{\pi}{2} \Rightarrow \frac{2L}{\lambda} = (2n+1) \times \frac{1}{2}$$

$$\Rightarrow \qquad \frac{2L\nu}{v} = (2n+1)\frac{1}{2}$$

$$\boxed{\text{Frequency of the vibration, } \nu = \frac{(2n+1)}{4L} v}$$

where, $n = 0, 1, 2$

where, ν is frequency and v is the speed of the wave.

The above frequencies are the normal frequencies of vibration. The fundamental frequency is obtained when $n = 0$.

$$\nu_0 = \frac{v}{4L} \qquad \text{[first harmonic]}$$

The overtone frequencies are

$$\nu_1 = \frac{3v}{4L} = 3\nu_0 \qquad \text{[third harmonic]}$$

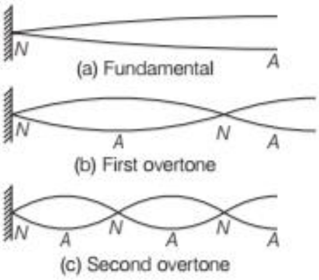
$$\nu_2 = \frac{5v}{4L} = 5\nu_0 \qquad \text{[fifth harmonic]}$$

$$\nu_3 = \frac{7v}{4L} = 7\nu_0 \qquad \text{[seventh harmonic]}$$

$$\nu_0 : \nu_1 : \nu_2 : \nu_3 \dots = 1:3:5:7:\dots$$

Here, we see that only odd harmonics are present. (i.e. contains odd multiples of the fundamental frequency)

The figure below shows shapes of the string.

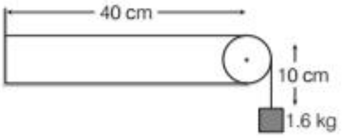


Laws of Transverse Vibration of String

- For a given string under a tension (T), the frequency of the vibrating string is inversely proportional to the vibrating length (L) of the string.
  i.e. $\nu \propto \frac{1}{L}$
- The frequency of the uniform string of given length is proportional to square root of tension (T) in the string
  i.e. $\nu \propto \sqrt{T}$
- For a given vibrating length and tension of the string, the frequency of the vibrating string is inversely proportional to the square root of the mass per unit length of the string.
  i.e. $\nu \propto \frac{1}{\sqrt{\mu}}$
- The fundamental frequency of a vibrating string is determined by a device known as sonometer.
- The laws of vibration of a string are also verified by using sonometer.

EXAMPLE |3| Fundamental Frequency

A 50 cm long wire of mass 20 g supports a mass of 1.6 kg as shown in figure. Find the fundamental frequency of the portion of the string between the wall and the pulley. Take, $g = 10\ \text{m/s}^2$.



Sol. Here, mass/length; $m = \dfrac{20 \times 10^{-3}}{50/100} = 0.04$ kg/m

$$T = 1.6 \text{ kg} = 1.6 \times 10 \text{ N} = 16 \text{ N}$$

Length that vibrates, $L = 50 - 10 = 40$ cm $= 0.4$ m

$$\therefore\ \nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.04}\sqrt{\frac{16}{0.04}} = \frac{20}{0.8} = 25 \text{ Hz}$$

EXAMPLE |4| Identical Wires

Two identical wires of length L and $2L$ vibrate with fundamental frequencies 100 Hz and 150 Hz, respectively. What is the ratio of their tensions?

Sol. Let, $\mu_1 = \mu_2 = \mu$

$$\nu_1 = 100 = \frac{1}{2L}\sqrt{\frac{T_1}{\mu}}$$

$$\nu_2 = 150 = \frac{1}{2(2L)}\sqrt{\frac{T_2}{\mu}}$$

$$\frac{\nu_1}{\nu_2} = \frac{100}{150} = 2\sqrt{\frac{T_1}{T_2}}$$

$$\frac{T_1}{T_2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$T_2 = 9\,T_1$$

VIBRATIONS OF AIR COLUMN

The vibrating air column in organ pipes is a common example of stationary waves. An organ pipe is a cylindrical tube which may be closed at one end (closed organ pipe) or open at both ends (open organ pipe).

If the air in pipe at its open end is made to vibrate, longitudinal wave is produced. This wave travels along the pipe towards its far end and is reflected back. Thus, due to superposition of **incident** and **reflected waves,** stationary waves are formed in pipe.

Closed Organ Pipe

Now, consider normal modes of oscillation of an air column with one end closed and the other open (i.e. closed organ pipe). A glass tube partially filled with water illustrates this system. The end in contact with water is a node having maximum pressure change, while the open end is an antinode having least pressure change.

If we are taking the end in contact with water to be $x = 0$, the other end, $x = L$ is an antinode.

In this case, we will have

$$|\sin kx| = 1 \Rightarrow |\sin kL| = 1$$

$$\Rightarrow \quad kL = \left(n + \frac{1}{2}\right)\pi$$

$$\Rightarrow \quad L = (2n+1)\frac{\lambda}{4};\ n = 0, 1, 2, 3, \ldots \left[\because k = \frac{2\pi}{\lambda}\right]$$

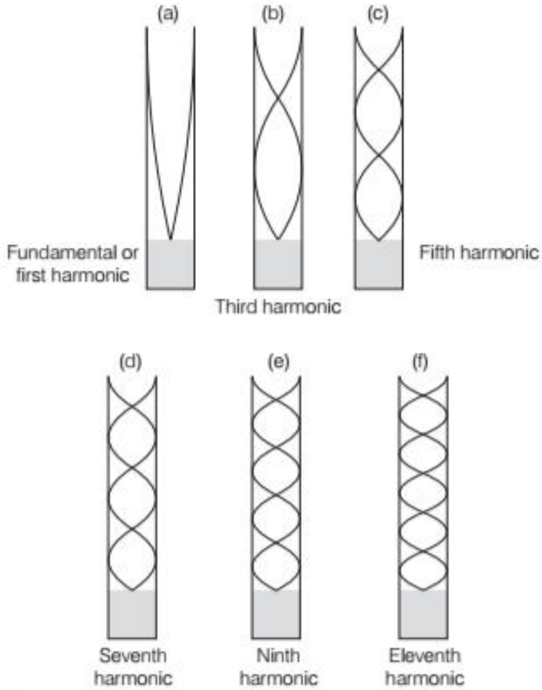
$$\Rightarrow \quad \boxed{\text{Frequency, } \nu = (2n+1)\frac{v}{4L};}\ n = 0, 1, 2, 3, \ldots$$

For fundamental frequency, $n = 0$ and $\nu = \dfrac{v}{4L}$

The higher frequencies are odd harmonics i.e. $\dfrac{3v}{4L}, \dfrac{5v}{4L}$, etc.

Thus, in a closed end pipe, only odd harmonics are present.

Figure below shows odd harmonics of air column with one end closed and the other is open.



Note

- If the external frequency is close to one of the natural frequencies, the system is said to be in **resonance.**
- Normal modes of a circular membrane rigidly clamped to the circumference are determined by boundary condition that no point on the circumference of the membrane vibrates.
- The standing waves in closed organ pipe is similar to the standing wave on a string fixed at one end.

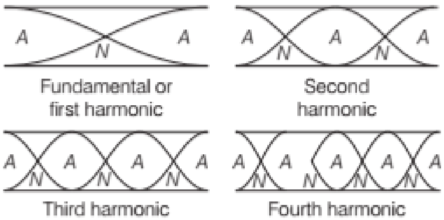
Open Organ Pipe

For a pipe opened at both ends, each (i.e. an open organ pipe) end is an antinode. It is then easily seen that an open air column at both ends generates all harmonics. The equation for the frequency will be same as that of string fixed at both ends.

$$\boxed{\text{Frequency, } \nu = \frac{nv}{2L}} \quad \text{for } n = 1, 2, 3, \dots$$

At both ends, antinodes will present and nodes will be alternate to antinodes.

The diagram is shown below



Fundamental or first harmonic; Second harmonic; Third harmonic; Fourth harmonic

For $n = 1$, $\quad \nu_1 = \dfrac{v}{2L}$

[fundamental frequency or first harmonic]

For $n = 2$, $\quad \nu_2 = \dfrac{v}{L} = 2\nu_1$

[second harmonic, first overtone]

For $n = 3$, $\quad \nu_3 = \dfrac{3v}{2L} = 3\nu_1$

[third harmonic, second overtone]

For $n = 4$, $\quad \nu_4 = \dfrac{4v}{2L} = 4\nu_0$

[fourth harmonic, third overtone]

$$\nu_1 : \nu_2 : \nu_3 : \dots = 1 : 2 : 3 : \dots$$

Note

- The sound produced by open organ pipe is more shriller than that produced by closed organ pipe.
- The quality of sound produced by an open organ pipe is better than produced by the closed organ pipe.
- The standing waves formed in open organ pipe is similar to standing waves formed on a string fixed at both ends.

EXAMPLE |5| Open and Closed Flute

A pipe 30.0 cm long is opened at both ends. Which harmonic mode of the pipe resonates with a 1.1 kHz source? Will resonance with the same source be observed, if one end of the pipe is closed? Take the speed of sound in air as 330 ms^{-1}. **[NCERT]**

Sol. Here, $L = 30.0 \text{ cm} = 0.3 \text{ m}$

Let nth harmonic of open pipe resonate with 1·1 kHz source,

i.e. $\quad \nu_n = 1.1 \text{ kHz} = 1100 \text{ Hz}$

As, $\quad \nu_n = \dfrac{nv}{2L}$

$\therefore \quad n = \dfrac{2L\nu_n}{v} = \dfrac{2 \times 0.30 \times 1100}{330} = 2$

i.e. **2nd harmonic** resonates with open pipe.

If one end of pipe is closed, its fundamental frequency put $n = 0$ in $\nu = (2n+1)\dfrac{v}{4L}$

$$\nu_1 = \frac{v}{4L} = \frac{330}{4 \times 0.3} = 275 \text{ Hz}$$

As odd harmonics alone are produced in a closed organ pipe, therefore, possible frequencies are $3\nu_1 = 3 \times 275 = 825$ Hz, $5\nu_1 = 5 \times 275 = 1375$ Hz and so on. As the source frequency is 1100 Hz, therefore, **no resonance** can occur when the pipe is closed at one end.

BEATS

This phenomenon arises from interference of waves having nearly same frequencies. The periodic variation in the intensity of sound wave caused by the superposition of two sound waves of nearly same frequencies and amplitude travelling in the same direction are called **beats**.

One rise and one fall in the intensity of sound constitutes **one beat** and the number of beats per second is called **beat frequency**.

The frequency of two sources or two waves should not differ by more than 10 Hz because, if it is more than 10 Hz, then it becomes difficult to distinguish between rise and fall in intensity of sound due to persistence of hearing.

Tuning the Instruments

Artists use the phenomenon of beat while tuning their instruments with each other. They go on tuning until their sensitive ears do not detect any beats. In this way, they match the frequencies of different instruments of the band.

Analytical Method of Beats

Let us consider two harmonic sound waves of nearly equal angular frequency ω_1 and ω_2 and suppose we are concerned with $y = 0$ only. Then, we can write the equations of waves as

$$y_1 = a \cos \omega_1 t \text{ and } y_2 = a \cos \omega_2 t$$

Here, we are considering sound wave hence, waves are represented by y_1, y_2 to show longitudinal displacement.

Now, by principle of superposition, the resultant displacement (y) is

$$y = y_1 + y_2 = a[\cos \omega_1 t + \cos \omega_2 t]$$
$$= 2a \cos \frac{(\omega_1 - \omega_2)t}{2} \cos \frac{(\omega_1 + \omega_2)t}{2}$$

$$= 2a \cos \omega_b t \cos \omega_a t$$

where, $\omega_b = \dfrac{\omega_1 - \omega_2}{2}$ and $\omega_a = \dfrac{\omega_1 + \omega_2}{2}$

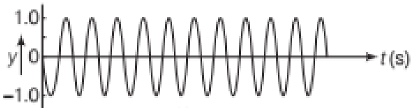
Now, if we assume $|\omega_1 - \omega_2| << \omega_1, \omega_2$ which means $\omega_a >> \omega_b$. This can be interpreted as the resultant wave is oscillating with the average angular frequency ω_a. The amplitude will be maximum when $|\cos \omega_b t|$ is 1.

Hence, the resultant wave, waxes and wanes with a frequency which is $2\omega_b = \omega_1 - \omega_2$.

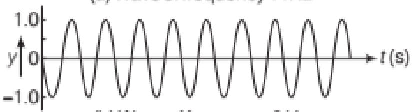
Hence, beat frequency,

$$\boxed{\text{Beat frequency, } \nu_{\text{beat}} = \nu_1 - \nu_2.}$$

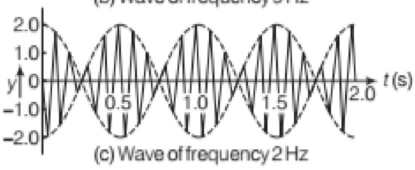
The figures below illustrate the phenomenon of beats for two harmonic waves of frequencies 11 Hz and 9 Hz. The amplitude of the resultant wave shows beats at a frequency of 2 Hz.



(a) Wave of frequency 11Hz



(b) Wave of frequency 9 Hz



(c) Wave of frequency 2 Hz

> **Musical Pillars**
>
> Temples often have some pillars. The rock used produces basic notes.
>
> Musical pillars are categorised into three types.
>
> Shruti pillar It can produce the basic note 'swaras'.
>
> Ganga Thoongal It produces the basic tunes which make up the 'ragas'.
>
> Laya Thoongal It produces 'taal' (beats) when tapped.

EXAMPLE |6| An Amazing Sitar

Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B, if the frequency of A is 427 Hz? **[NCERT]**

Sol. Here, frequency of $A = 427$ Hz

As number of beats/sec $(m) = 5$ Hz

$\therefore$ Possible frequencies of B are (427 ± 5) Hz $= 432$ Hz or 422 Hz.

When tension of B is increased, its frequency increases. Number of beats/s decreases to 3.

Therefore, m is negative.

Hence, original frequency of $B = 427 - 5 = 422$ Hz.

EXAMPLE |7| Identical Piano Strings

Consider the two identical piano strings, each tuned exactly to the 420 Hz. The tension in any one of the strings is increased by 2.0%. If they are now struck, what is the beat frequency between the fundamentals of the two strings? Take, length of the strings $= 65$ cm.

As the tension in one of the strings is changed, its fundamental frequency changes. Therefore, when both strings are played, they will have different frequencies and beats will be heard.

Sol. If ν_1, v_1, T_1 and ν_2, v_2, T_2 are the frequencies, velocity and tension in the first and second strings, respectively, then

$$\frac{\nu_2}{\nu_1} = \frac{v_2/2L}{v_1/2L} = \frac{v_2}{v_1} \Rightarrow \frac{\nu_2}{\nu_1} = \frac{\sqrt{T_2/\mu}}{\sqrt{T_1/\mu}} = \sqrt{\frac{T_2}{T_1}}$$

Given that the tension in one string is 2.0% larger than the other

i.e. $\quad T_2 = T_1 + \dfrac{2}{100} T_1 = 1.02\, T_1$

The ratio of frequencies $\dfrac{\nu_2}{\nu_1} = \sqrt{\dfrac{1.02\, T_1}{T_1}} = 1.01$ Hz

Now, solve for the frequency of the tightened string.

$$\nu_2 = 1.01\,\nu_1 = 1.01 \times 420 = 424.2 \text{ Hz}$$

[given, $\nu_1 = 420$ Hz]

Thus, the beat frequency,

$$\nu_{\text{beat}} = \nu_2 - \nu_1 = 424.2 - 420 = 4.2 \text{ Hz}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. A standing wave is generated on a string. Which of the following statement(s) is/are correct for the standing waves?

(a) The amplitude of standing wave varies from point to point but each element of the string oscillates with the same angular frequency 'ω' or time period

(b) The string as a whole vibrates in phase with differing amplitudes at different points

(c) The wave pattern in neither moving to the right nor to the left

(d) All of the above

Sol. (d) There is no phase difference between oscillations of different elements of the wave. However, the string as a whole vibrates in phase with different amplitudes at different points. Also, there is zero movement of the wave pattern. Hence, they are called standing or stationary waves.

2. Let a wave $y(x,t) = a\sin(kx - \omega t)$ is reflected from an open boundary and then the incident and reflected waves overlaps. Then the amplitude of resultant wave

(a) $2a\cos(kx)$ (b) $2a\sin(kx)$

(c) $2a\sin\left(\frac{kx}{2}\right)$ (d) $a\sin(kx)$

Sol. (b) We have incident wave $y_1 = a\sin(kx - \omega t)$

So the reflected wave is $y_2 = a\sin(kx + \omega t)$

From principle of superposition, the standing wave equation obtained after superimposing y_1 and y_2, we get

$$y(x, t) = 2a\sin kx \cos\omega t$$

Thus, the resultant amplitude is

$$A(x) = 2a\sin kx$$

3. At nodes in stationary waves

(a) change in pressure and density are maximum

(b) change in pressure and density are minimum

(c) strain is zero

(d) energy is maximum

Sol. (b) In stationary waves, all particles except nodes oscillate with same frequency but amplitude is zero at nodes and maximum at anti-nodes. Thus, change in pressure and density is minimum at nodes.

4. Following two wave trains are approaching each other.

$$y_1 = a\sin 2000\,\pi t$$

$$y_2 = a\sin 2008\,\pi t$$

The number of beats heard per second is

(a) 8 (b) 4

(c) 1 (d) zero

Sol. (b) Beat frequency $= f_2 - f_1 = \frac{\omega_2 - \omega_1}{2\pi}$

$$= \frac{2008\pi - 2000\pi}{2\pi} = 4\text{ Hz}$$

VERY SHORT ANSWER Type Questions

5. Why should the difference between the frequencies be less than 10 to produce beats?

Sol. Human ear cannot identify any change in intensity is less than $\left(\frac{1}{10}\right)$th of a second. So, difference should be less than 10.

6. In a hot summer day, pitch of an organ pipe will be higher or lower?

Sol. The speed of sound in air is more at higher temperature, as $v \propto \sqrt{T}$. As we know, frequency $\nu = \frac{v}{\lambda}$ as v is more, hence, ν will be more and accordingly pitch will be more.

7. Show that when a string fixed at its two ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops, the frequencies are in the ratio 1 : 2 : 3 : 4. **[NCERT Exemplar]**

Sol. In case of a string fixed at two ends, when the string vibrates in n loops

$$\nu_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow \nu_n \propto n$$

Hence, when the string vibrates in 1 loop, 2 loops, 3 loops, 4 loops, the frequencies are in the ratio 1 : 2 : 3 : 4.

8 A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 kms^{-1}? The operating frequency of the scanner is 4.2 MHz. **[NCERT]**

Sol. $v = 1.7$ km/s $= 1700$ m/s, $\nu = 4.2$ MHz $= 4.2 \times 10^6$ Hz

$$\lambda = \frac{v}{\nu} = \frac{1700}{4.2 \times 10^6}$$

$$= 0.405 \times 10^{-3}\text{ m} = 0.405\text{ mm}$$

$$= 4.1 \times 10^{-4}\text{ m}$$

9. When two waves of almost equal frequencies n_1 and n_2 reach at a point, simultaneously. What is the time interval between successive maxima? **[NCERT Exemplar]**

Sol. Number of beats/s $= (n_1 - n_2)$

Hence, time interval between two successive beats = time interval between two successive maxima $= \frac{1}{n_1 - n_2}$

10. A sonometer wire is vibrating in resonance with a tuning fork. Keeping the tension applied same, the length of the wire is doubled. Under what conditions would the tuning fork still be in resonance with the wire? **[NCERT Exemplar]**

Sol. The sonometer frequency is given by

$$\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

Now, as it vibrates with length L, we assume $\nu = \nu_1$

$$n = n_1$$

$$\therefore \quad \nu_1 = \frac{n_1}{2L}\sqrt{\frac{T}{\mu}} \quad \text{...(i)}$$

When length is doubled, then $\nu_2 = \frac{n_2}{2 \times 2L}\sqrt{\frac{T}{\mu}}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\nu_1}{\nu_2} = \frac{n_1}{n_2} \times 2$$

To keep the resonance, $\frac{\nu_1}{\nu_2} = 1 = \frac{n_1}{n_2} \times 2$

$\Rightarrow \qquad n_2 = 2n_1$

Hence, when the wire is doubled, the number of loops also get doubled to produce the resonance. That is, it resonates in second harmonic.

SHORT ANSWER Type Questions

11. A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected. **[NCERT]**

Sol. As, there is piston at one end, it behaves as a closed organ pipe. Hence, it will produce odd harmonics only.

Hence, resonant frequencies will be first and third harmonic.

In the fundamental mode, $\frac{\lambda}{4} = 25.5$ cm

$\Rightarrow \qquad \lambda = 4 \times 25.5 = 102 \text{ cm} = 1.02 \text{ m}$

Speed of sound in air

$$v = \nu\lambda = 340 \times (1.02) = 346.8 \text{ m/s}$$

12. A tuning fork *A*, marked 512 Hz, produces 5 beats per sec, where sounded with another unmarked tuning fork *B*. If *B* is loaded with wax, the number of beats is again 5 per sec. What is the frequency of the tuning fork *B* when not loaded? **[NCERT Exemplar]**

Sol. Frequency of *A*, $\nu_0 = 512$ Hz

Number of beats/s = 5

Frequency of $B = 512 \pm 5 = 517$ or 517 Hz

On loading its frequency decreases from 517 to 507, so that number of beats/s remain 5.

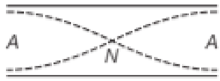
Hence, frequency of *B* when not loaded = 517 Hz.

13. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel? **[NCERT]**

Sol. Given, $L = 100 \text{ cm} = 1 \text{ m}$,

$$\nu = 2.53 \text{ kHz} = 2.53 \times 10^3 \text{ Hz}$$

As the given rod is clamped at middle, hence, there will be a node at the middle..



Hence, length, $L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$

$\Rightarrow \qquad \lambda = 2L = 2 \text{ m}$

Speed, $\qquad v = \nu\lambda = 2.53 \times 10^3 \times 2 = 5.06 \times 10^3 \text{ ms}^{-1}$

14. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a source of 1237.5 Hz? (sound velocity in air = 330 ms^{-1}) **[NCERT Exemplar]**

Sol. Length of pipe $(l) = 20 \text{ cm} = 20 \times 10^{-2}$ m

$$\nu_{\text{funda}} = \frac{v}{4L} = \frac{330}{4 \times 20 \times 10^{-2}}$$

$$\nu_{\text{funda}} = \frac{330 \times 100}{80} = 412.5 \text{ Hz}$$

$$\frac{\nu_{\text{given}}}{\nu_{\text{funda}}} = \frac{1237.5}{412.5} = 3$$

Hence, 3rd harmonic mode of the pipe is resonantly excited by the source of given frequency.

15. (i) For the wave on a string described by $Y = 0.06 \sin 2\pi/3x \cos(120\pi t)$, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers.

(ii) What is the amplitude of a point 0.375 m away from one end? **[NCERT]**

Sol. (i) All the points except the nodes on the string have the same frequency and phase but not the same amplitude.

(ii) Given, $Y = 0.06 \sin \frac{2\pi}{3} x \cos(120\pi t)$

Putting $x = 0.375$ m

Amplitude, $Y = 0.06 \sin \frac{2\pi}{3} \times (0.375)$

$$= 0.06 \sin \frac{\pi}{4} = \frac{0.06}{\sqrt{2}} = 0.042 \text{ m}$$

16. A narrow sound pulse (e.g. a short pip by a whistle) is sent across a medium.

(i) Does the pulse have a definite (a) frequency, (b) wavelength, (c) speed of propagation?

(ii) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz? **[NCERT]**

Sol. (i) A short pip by a whistle

(a) will not have a fixed frequency.

(b) will not have fixed wavelength.

(c) will have definite speed that will be equal to speed of sound in air.

(ii) 0.05 Hz will be the frequency of repetition of the short pip.

17. A sitar wire is replaced by another wire of same length and material but of three times the earlier radius. If the tension in the wire remains the same, then by what factor will the frequency change? **[NCERT Exemplar]**

Sol. $\nu_1 = \dfrac{1}{l_1 D_1}\sqrt{\dfrac{T_1}{\pi\rho_1}}$

where, D = diameter of wire

$$\nu_2 = \frac{1}{l_2 D_2}\sqrt{\frac{T_2}{\pi\rho_2}}$$

$$l_1 = l_2, \rho_2 = \rho_1$$

$$T_2 = T_1, D_2 = 3D_1$$

$$\Rightarrow \quad \nu_2 = \frac{\nu_1}{3}$$

New frequency is $\frac{1}{3}$rd of the original frequency.

18. Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, then what is the frequency of B? **[NCERT]**

The difference in frequencies is the number of beats. If tension in the wire is increased, then the frequency is also increased and *vice-versa*.

Sol. Given, frequency of A, $f_A = 324$ Hz

Now, frequency of B, $f_B = f_A \pm$ beat frequency

$$= 324 \pm 6$$

or $$f_B = 330 \text{ or } 318 \text{ Hz}$$

Now, if tension in the string is slightly reduced, its frequency will also reduce from 324 Hz.

Now, if $f_B = 330$ and f_A reduces, then beat frequency should increase which is not the case but if $f_B = 318$ Hz and f_A decreases, the beat frequency should decrease, which is the case and hence, $f_B = 318$ Hz.

19. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kgm^{-1}. What is (i) the speed of a transverse wave on the string and (ii) the tension in the string? **[NCERT]**

Sol. Here, given $\nu = 45$ Hz, $M = 3.5 \times 10^{-2}$ kg

$$\mu = \frac{\text{Mass}}{\text{Length}} = 4.0 \times 10^{-2} \text{ kgm}^{-1}$$

$$l = \frac{M}{\mu} = \frac{3.5 \times 10^{-2}}{4 \times 10^{-2}} = \frac{7}{8} \text{ m}$$

$$l = \frac{\lambda}{2} = \frac{7}{8} \Rightarrow \lambda = \frac{7}{4} \text{ m} = 1.75 \text{ m}$$

(i) Speed, $v = \nu \times \lambda = 45 \times 1.75 = 78.75$ m/s

(ii) As $v = \sqrt{\dfrac{T}{\mu}} \Rightarrow T = v^2 \times \mu$

$$\Rightarrow T = (78.75)^2 \times 4 \times 10^{-2}$$

$$\Rightarrow T = 248.06 \text{ N}$$

20. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, then what is the wavelength of (i) the reflected sound, (ii) the transmitted sound? Speed of sound in air is 340 ms^{-1} and in water 1486 ms^{-1}. **[NCERT]**

Sol. Given, $\nu = 1000 \text{ kHz} = 10^6$ Hz

$$v_a = 340 \text{ m/s}, v_w = 1486 \text{ m/s}$$

Wavelength of reflected sound,

$$\lambda_a = \frac{v_a}{\nu} = \frac{340}{10^6} = 3.4 \times 10^{-4} \text{ m}$$

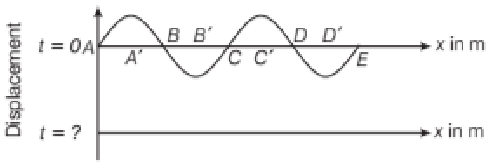
Wavelength of transmitted sound,

$$\lambda_w = \frac{v_s}{\nu} = \frac{1486}{10^6} = 1486 \times 10^{-6}$$

$$\lambda_w = 1.486 \times 10^{-3} \text{ m}$$

LONG ANSWER Type I Questions

21. The pattern of standing waves formed on a stretched string at two instants of time are shown in figure. The velocity of two waves superimposing to form stationary waves is 360 m/s and their frequencies are 256 Hz.



(i) Calculate the time at which the second curve is plotted.

(ii) Mark nodes and antinodes on the curve.

(iii) Calculate the distance between A' and C'.

[NCERT Exemplar]

Sol. Here, $v = 360$ m/s, $\nu = 256$ Hz

(i) In the figure, second curve represents all the particles passing simultaneously through mean position. This happens at $t = \frac{T}{4}\left(T = \frac{1}{256}\text{s}\right)$

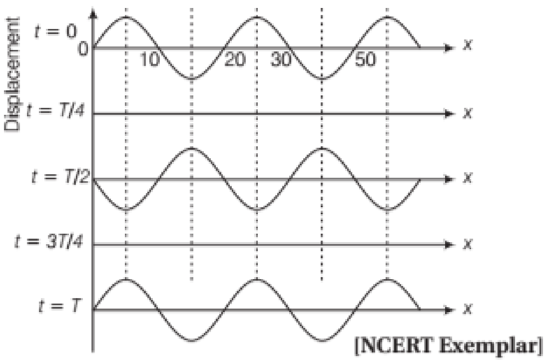
$$= \frac{1}{4 \times 256}\text{ s} = 9.8 \times 10^{-4}\text{ s}$$

(ii) Nodes are at points A, B, C, D, E and antinodes are at A', B', C', D'

(iii) Distance between A' and C'

$$= \frac{2\lambda}{2} = \lambda = \frac{v}{\nu} = \frac{360}{256} = 1.41\text{ m}$$

22. The wave pattern on a stretched string is shown in figure. Interpret what kind of wave this is and find its wavelength?



[NCERT Exemplar]

Sol. The wave pattern shown represent stationary wave.

t	x (cm)
0	0
$\frac{T}{2}$	10
T	20

At the above time and position and similarly for next interval and position. These points are at rest and called nodes.

Distance between successive nodes $= \frac{\lambda}{2} = 10$ cm

$\Rightarrow \qquad \lambda = 20$ cm

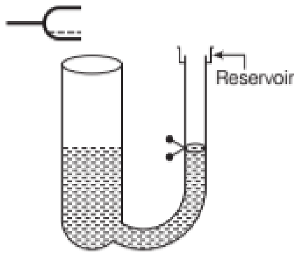
23. A tuning fork vibrating with a frequency of 512 Hz is kept close to the open end of a tube filled with water shown in figure. The water level in the tube is gradually lowered. When the water level is 17 cm below the open end, maximum intensity of sound is heard.

If the room temperature is 20°C, then calculate

(i) speed of sound in air at room temperature.

(ii) speed of sound in air at 0°C.

(iii) if the water in the tube is replaced with mercury, will there be any difference in your observations? **[NCERT Exemplar]**



Sol. (i) $\nu = \frac{v}{4l}$

$$\Rightarrow v = (\nu)(4l) = (512)\left(\frac{4 \times 17}{100}\right)\text{ m/s} = 348.16\text{ m/s}$$

(ii) $v_0 = v_t\sqrt{\frac{T_0}{T}} = 348.16\sqrt{\frac{273}{273+20}} = 336.1$ m/s

(iii) There will not be any change in resonance length, as frequency will remain same. As mercury acts as reflector intensity may change to some extent.

24. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 ms^{-1}) **[NCERT]**

Sol. Given, $L = 20$ cm $= 0.2$ m, $\nu_n = 430$ Hz, $v = 340$ m/s

It will behave as closed organ pipe

$$\nu_n = (2n-1)\frac{v}{4L}, \text{ where } n = 1, 2, 3, \ldots$$

$$\Rightarrow \quad 430 = (2n-1)\frac{v}{4L} = (2n-1) \times \frac{340}{4 \times 0.2}$$

$$\Rightarrow (2n-1) = \frac{430 \times (0.8)}{340} \Rightarrow 2n = \frac{(430)(0.8)}{340} + 1$$

$$\Rightarrow \quad n = \frac{43 \times 4}{340} + \frac{1}{2} = \frac{2 \times 172 + 340}{340 \times 2} = \frac{684}{680} = 1.006$$

Hence, it will be the 1st normal mode or harmonic mode of vibration.

In a pipe open at both ends, $\nu_n = n \times \frac{v}{2l} = \frac{n \times 340}{2 \times 0.2} = 430$

$$\Rightarrow \quad n = \frac{430 \times 2 \times 0.2}{340} = \frac{43 \times 2 \times 2}{340} = 0.5$$

As n is not integer, hence, open organ pipe cannot be in resonance with the source.

LONG ANSWER Type II Questions

25. Explain why (or how)

(i) in a sound wave, a displacement node is a pressure antinode and *vice-versa*?

(ii) bats can ascertain distances, directions, nature and sizes of the obstacles without any eyes?

(iii) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes?

(iv) solids can support both longitudinal and transverse waves but only longitudinal waves can propagate in gases, and

(v) the shape of a pulse gets distorted during propagation in a dispersive medium? [NCERT]

Sol. (i) **Node** It is a point where the amplitude of oscillation is zero, i.e. displacement is minimum. As pressure is inversely related with displacement, i.e. when displacement will be minimum, pressure will be maximum.

Antinode At this point displacement is maximum, i.e. amplitude of oscillation will be maximum and hence, pressure will be minimum as it is inversely related.

(ii) Bats emit ultrasonic waves of large frequencies. These waves will be reflected by the obstacles in their path. The reflected rays received by the bat will give idea about the obstacle, i.e. distance, direction, size and nature.

(iii) As overtones produced and relative strengths of notes are different in two notes of violin and sitar.

Although frequencies are same, we will distinguish by their strengths.

(iv) The reason behind is that solids have both the elasticity of volume as well as shape, whereas gases have only the volume elasticity.

(v) As in the dispersive medium wavelengths are different, hence the velocities, therefore, the shape of the pulse gets distorted.

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. To increase the frequency from 100 Hz to 400 Hz the tension in the string has to be changed by

(a) 4 times (b) 16 times
(c) 2 times (d) None of these

2. A resonating column has resonant frequencies as 100 Hz, 300 Hz, 500 Hz. Then it may

(a) an open pipe
(b) a pipe closed at both ends
(c) pipe closed at one end
(d) Data insufficient

3. When two harmonic sound waves of close (but not equal) frequencies are heard at the same time, we hear

(a) a sound of similar frequency
(b) a sound of frequency which is the average of two close frequencies
(c) audibly distinct waxing and waning of the intensity of the sound with a frequency equal to the difference in the two close frequencies
(d) All of the above

4. A person blows into an open end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

(a) a high pressure pulse starts travelling up the pipe, if the other end f the pipe is open
(b) a low pressure pulse starts travelling up the pipe, if the other end of the pipe is open
(c) a low pressue pulse starts travelling down the pipe, if the other end of the pipe is closed
(d) None of the above

Answer

1. (b) | 2. (c) | 3. (d) | 4. (b)

VERY SHORT ANSWER Type Questions

5. When are standing waves formed? Of the closed and open organ pipes, which one will produce better musical sound and why?

6. What are echo and acoustics?

7. In case of a stationary wave, where will a man hear maximum sound, at the node or at the antinode?

8. A string vibrates according to the equation $y = 5\sin\frac{\pi x}{3}\cos 40\pi t$, where x and y are in centimetres and t is in second. What is the speed of the component wave? [**Ans.** 120 cm/s]

9. Two tuning forks of frequencies 250 Hz and 252 Hz are being vibrated simultaneously. If a loud sound is produced just now, after what time would the sound be again equally loud?

SHORT ANSWER Type Questions

10. Third overtone of a closed organ pipe is in unison (resonance) with fourth harmonic of an open organ pipe. Find the ratio of lengths of the pipes. [**Ans.** 7 : 8]

11. What are beats? When are beats formed? What is their frequency?

12. By drawing figures, show the formation of first four harmonic setup in a vibrating string or wire fixed at its two ends. Mark nodes and antinodes.

LONG ANSWER Type I Questions

13. What do you mean by the terms, overtones and harmonics? Briefly explain. Find frequency of first harmonics for a string fixed at one end.

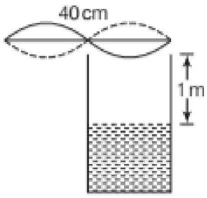
14. A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 180 Hz?
[**Ans.** 60 cm]

15. A standing wave is formed by two harmonic waves, $Y_1 = A\sin(kx - \omega t)$ and $Y_2 = A\sin(kx + \omega t)$ travelling on a string in opposite directions. Mass density of the string is ρ and area of cross-section is s. Find the total mechanical energy between two adjacent nodes on the string.
[**Ans.** $\frac{\rho A^2 \omega^2 \pi s}{k}$]

LONG ANSWER Type II Questions

16. Discuss formation of different modes of vibration in (i) an open end, (ii) a closed end air column.
Draw neat diagram also.

17. A wire of length 40 cm which has a mass of 4 g oscillates in its second harmonic and sets the air column in the tube to vibration in its fundamental mode as shown in figure. $v = 340$ m/s for sound.
Find the tension in the wire. [**Ans.** 11.57 N]



SUMMARY

- **Wave motion** It is a kind of disturbance which travels through a medium due to the repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next.
- Three basic types of waves
  (i) Mechanical waves (ii) Electromagnetic waves (iii) Matter waves
- **Transverse waves** These are the waves in which particles of the medium vibrate about their mean positions in a direction perpendicular to the direction of propagation of the disturbance. These waves can propagate in those media which have a shear modulus of elasticity. e.g. Solids.
- **Longitudinal waves** These are the waves in which particles of the medium vibrate about their mean positions along the direction of propagation of the disturbance. These waves can propagate in those media having a bulk modulus of elasticity and are therefore, possible in all media: solids, liquids and gases.
- **Progressive wave** A wave that moves from one point of medium to another is called a progressive wave.
- **Displacement relation in a progressive wave** $y(x, t) = a\sin(kx - \omega t + f)$ for +ve x-direction.
  $y(x, t) = a\sin(kx - \omega t + f)$ for –ve x-direction.
- **Amplitude (A)** It is the maximum displacement suffered by the particles of the medium from the mean position during the ropagation of a wave.
- **Time period (T)** It is the time in which a particle of the medium completes one vibration about its mean position.
- **Frequency (ν)** It is the number of waves produced per second in a given medium.
- **Wavelength (λ)** It is the distance covered by a wave during the time a particle of the medium completes one vibration about its mean position. It is the distance between two nearest particles of the medium which are vibrating in the same phase.
- **Characteristics of waves** Wavelength and angular wave number $\lambda = \frac{2\pi}{k}$
- Period, angular frequency and frequency $\omega = \frac{2\pi}{T}, \nu = \frac{1}{T}$
- Speed of a travelling wave $v = \nu\lambda = \frac{\lambda}{T}$

- Transverse wave on stretched spring $v = \sqrt{T/\mu}$
- **Principle of superposition of waves** When two or more pulses overlap, the resultant displacement is the algebraic sum of the displacements due to each pulse $y'(x, t) = y_1(x, t) + y_2(x, t)$
- **Interference of waves** The phenomenon that occurs when the two waves meet while travelling along the same medium.

$$y(x, t) = 2a\cos\frac{\phi}{2}\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

- If $f = 0$, two waves are in phase $y(x, t) = 2a\sin(kx - \omega t)$
- If $\phi = \pi$, two waves are out of phase by 180°,

$$y(x, t) = 0$$

- **Constructive interference** When waves interfere in phase $\phi = 2n\pi$, $\Delta x = n\lambda$
- **Destructive interference** When waves interfere in phase opposition $\phi = (2n+1)\pi$, $\Delta x = \dfrac{(2n+1)\lambda}{2}$
- **Vibrations of air column** Closed organ pipe, $v = (2n+1)\dfrac{v}{4L}$

  Open organ pipe, $v = \dfrac{nv}{2L}$
- **Relation between phase difference, path difference and time difference** Relation between phase difference, path difference and time difference is given by

  Phase difference $(\phi) = \dfrac{2\pi}{\lambda} \times$ Path difference (x)

  $\Rightarrow \phi = \dfrac{2\pi x}{\lambda} \Rightarrow x = \dfrac{\phi\lambda}{2\pi}$

  Speed of mechanical wave, $v = \sqrt{\dfrac{E}{\rho}}$

  Where E = The modulus of elasticity of the
  and ρ = The density of the medium
- **Speed of a transverse wave on stretched string**

$$v = \sqrt{\frac{T}{\mu}}$$

Where, T = Tension in the string and μ = Linear mass density

- **Velocity of longitudinal waves in elastic medium** Solid medium $v = \sqrt{\dfrac{Y}{\rho}}$

  Where, Y is the Young's modulus of elasticity and ρ is its density.
- **Liquid medium** $v = \sqrt{\dfrac{B}{\rho}}$

  Where, B is the Bulk modulus of liquid and ρ is its density.
- **Gas medium** $v = \sqrt{\dfrac{p}{\rho}}$ where, p is the pressure of gas and ρ is its density.

$$v_{Solid} > v_{Liquid} > v_{Gaseous}$$

- **Laplace's correction** He pointed out that the propagation of sound in gaseous medium is not an isothermal process but an adiabatic process. Thus,

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

  where, γ is the ratio of specific heat of the gas at constant pressure p to that at constant volume.
- **Beats** It is the difference of nearly same frequencies and amplitudes.

  Beat frequency, $v_{beat} = v_1 - v_2$

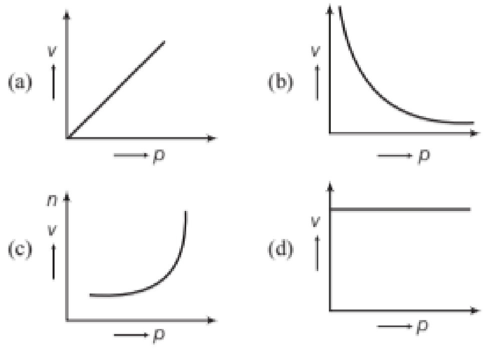
CHAPTER PRACTICE

OBJECTIVE Type Questions

1. The picture of a progressive transverse wave at a particular instant of time gives
(a) shape of the wave
(b) motion of the particle of the medium
(c) velocity of the wave
(d) None of the above

2. In a longitudinal wave, the elastic property of the constituents of the medium that determines the stress under compressional strain is
(a) Young's modulus (Y) (b) bulk modulus (B)
(c) shear modulus (S) (d) Either (b) or (c)

3. A student plotted the following four graphs representing the variation of velocity of sound in a gas with the pressure p at constant temperature. Which one is correct?



4. A wave equation is given by

$$y = 4 \sin \left[\pi \left(\frac{t}{5} - \frac{x}{9} + \frac{1}{6} \right) \right]$$

where, x is in cm and t is in second. The wavelength of the wave is
(a) 18 cm (b) 9 cm
(c) 36 cm (d) 6 cm

5. The equation of a progressive wave can be given by $y = 15 \sin (660\pi t - 0.02\pi x)$ cm. The frequency of the wave is
(a) 330 Hz (b) 342 Hz
(c) 365 Hz (d) 660 Hz

6. Two waves of equal amplitude A and equal frequency travel in the same direction in a medium. The amplitude of the resultant wave is
(a) 0 (b) A
(c) $2A$ (d) between 0 to $2A$

7. If a propagating wave meets a boundary which is not completely rigid or is an interface between two different elastic media, then which of the statements is/are correct?
(a) A part of the incident wave is reflected and a part is transmitted into the second medium
(b) The incident wave is completely reflected from the boundary
(c) Only part of the wave is reflected and the remaining part disappears
(d) None of the above

8. The air column in a pipe open at both ends is oscillating with certain frequency. Which of the given statement (s) is/are correct for the open air column at both ends?
(a) Each end of the pipe acts as an antinode
(b) An open air column at both ends generates all harmonics
(c) Each end of the pipe is a node
(d) Both (a) and (b)

ASSERTION AND REASON

Directions (Q.Nos. 9-14) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

9. **Assertion** The light emitted by stars, which are hundreds of light years away, reaches us through inter-stellar spaces even though the inter-stellar spaces are practically vacuum.
Reason Light is an electromagnetic wave and do not necessarily require a medium for propagation, they can even travel in vacuum.

10. **Assertion** The amplitude $A(\phi)$ of the resultant of the two right travelling waves given by equations

$y_1(x, t) = A \sin (kx - \omega t)$

and $y_2(x, t) = A \sin (kx - \omega t + \phi)$

is decreasing as ϕ increases from 0 to π.

Reason The amplitude of the resultant of the two waves is given by $A(\phi) = 2A \cos \phi/2$ which is decreasing for $0 \le \phi \le \pi$.

11. **Assertion** Transverse waves are possible in solids and strings (under tension) but not in fluids.

Reason Solids and strings have no zero shear modulus that is they can sustain shearing stress but fluids yield to shearing stress and hence they do not have shape of their own.

12. **Assertion** Longitudinal waves can be propagated through solids and fluids both.

Reason Solids as well as fluids have non zero bulk modulus, that is they can sustain compressive stress and longitudinal waves involve compressive stress (pressure).

13. **Assertion** Speed of sound is more in liquids and solids than gases.

Reason Liquids and solids have higher densities than gases.

14. **Assertion** Superposition of two harmonic waves, one of frequency 11 Hz and the other of frequency 9 Hz gives rise to beats of frequency 2 Hz.

Reason Harmonic waves of nearly equal frequencies interfere to give rise to beat having beat frequency, $\nu_{beat} = |\nu_1 - \nu_2|$.

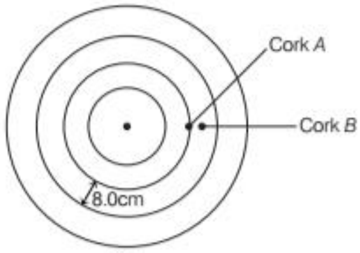
CASE BASED QUESTIONS

Directions (Q.Nos. 15-16) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

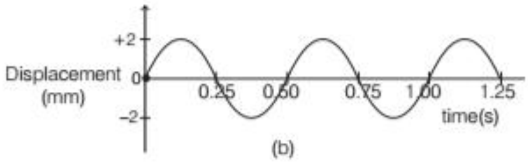
15. **Displacement of Wave**

A stone is dropped in a liquid at rest in a tank. The Fig. (a) below shows circular wave fronts. The waves produced at the centre of a circular ripple tank. Two corks A and B, floats on the water and moves up & down on the surface as the wave passes. The wavelength of the wave is 8.0 cm.

The Fig. (b) shows how the displacement of A varies with time.



(a)



(b)

(i) Name the type of waves produced on water surface.

(a) Longitudinal wave (b) Transverse wave
(c) Sound wave (d) EM wave

(ii) What is the amplitude of the vibrations of A as wave passes?

(a) 2 mm (b) 0.25 mm
(c) 0.50 mm (d) 8 mm

(iii) The horizontal distance between A and B is half the wavelength of the wave, then the displacement of B with time is

(a) same as that of A with equal magnitude
(b) opposite to that of A with equal magnitude
(c) double in magnitude as that of A
(d) half in magnitude as that of A

(iv) What is the frequency of the wave?

(a) 4 Hz (b) 0.4 Hz
(c) 2 Hz (d) 0.2 Hz

(v) If the distance between the centre of the ripple tank and its edge is 40 cm, then the time taken by the wave to travel from the centre of the tank to the edge is

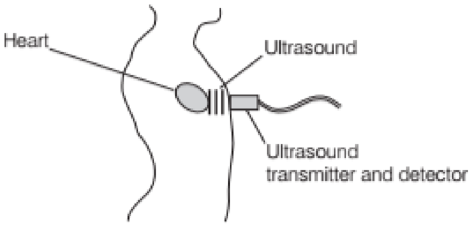
(a) 5 s (b) 2.5 s
(c) 3 s (d) 4.5 s

16. **Ultrasound**

Ultrasound is an example of longitudinal wave having high frequency. These types of waves are the one in which the individual particles of the medium execute simple harmonic motion about their mean positions along the direction of propagation of the wave.

Ultrasound is used to investigate the internal organs of a human body as it can penetrate into matter.

Figure below shows how ultrasound is used to produce an image of the heart.



(i) What is the frequency range of ultrasound?
(a) 20 Hz (b) 20 kHz
(c) Less than 20 Hz (d) More than 20 kHz

(ii) Which of these can produce ultrasound?
(a) Bats (b) Dolphins
(c) Porpoises (d) All of these

(iii) When ultrasound wave passes through a body, then the particles in the body have motion in
(a) forward and backward direction
(b) parallel to direction of motion
(c) perpendicular to direction of motion
(d) Both (a) and (b)

(iv) There are small bubbles of gas in a body. When the body is kept in the path of ultrasound, then
(a) bubbles gets expanded
(b) bubbles gets contracted
(c) Both (a) and (b)
(d) No change occur

(v) The ultrasound of wavelength 1.2×10^{-3} m has a speed of 1500 m/s in a human body. The frequency is
(a) 1.25×10^{-6} Hz (b) 1.25×10^{6} Hz
(c) 1.2×10^{-4} Hz (d) 2×10^{6} Hz

Answer

1. (a)	2. (b)	3. (d)	4. (a)	5. (a)
6. (d)	7. (a)	8. (d)	9. (a)	10. (a)
11. (a)	12. (a)	13. (b)	14. (a)	
15. (i) (b)	(ii) (a)	(iii) (b)	(iv) (c)	(v) (b)
16. (i) (d)	(ii) (d)	(iii) (d)	(iv) (c)	(v) (b)

VERY SHORT ANSWER Type Questions

17. Is an oscillation, a wave, why?

18. Give two examples of each of longitudinal and transverse waves.

19. What is the audible range of sound frequencies? [**Ans.** 20 Hz to 20 kHz]

20. Two sound waves produce 12 beats in 4 s. By how much do their frequencies differ? [**Ans.** 3 beats/s]

SHORT ANSWER Type Questions

21. In a resonance tube, the second resonance does not occur exactly at three times the length of first resonance, why?

22. Establish the relation, $v = \nu\lambda$ for a wave motion.

23. What is the direction of oscillations of the particles of a medium through which (i) transverse, (ii) longitudinal wave is propagating?

24. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far particles are two points 60° out of phase? [**Ans.** 0.12 m]

25. State few important use of phenomenon of beats.

LONG ANSWER Type I Questions

26. A tuning fork of frequency 200 Hz is in resonance with a sonometer wire. How many beats will be heard if tension in the wire is increased by 2%? [**Ans.** 2 beats]

27. What are stationary waves? Explain their formation analytically in case of a string fixed at both of its ends.

LONG ANSWER Type II Questions

28. State Newton's formula for velocity of sound in air. Point out the error and hence, discuss Laplace's correction and calculate the temperature at which the speed of sound will be two times of its value at 0°C. [**Ans.** 819°C]

29. Two tuning forks A and B give 5 beats/s. A resounds with a closed column of air 15 cm long and B with an open column of air 30.5 cm long.
Calculate their frequencies. Neglect and correction. [**Ans.** 305 Hz, 300 Hz]
Hint Use $m = v_1 - v_2$

30. Stationary waves are set up by the superposition of two waves given by $y_1 = 0.05\sin(5\pi t - x)$ and $y_2 = 0.05\sin(5\pi t + x)$, where x and y are in metre and t in second. Calculate the amplitude of a particle at a distance of $x = 1$ m. [**Ans.** 0.054 m]